Mechanics of Materials

Curated undergraduate study track (not accredited, no external exam). Original explanations and practice only, following the reviewed course handout.

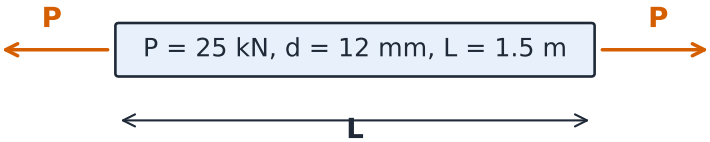
What the load does to the part

- The sixth course: stress, deformation and the failure conditions that decide whether a member is safe - computed on the bracket assembly the track has carried since Course 2.
- Steel members throughout: $E = 200$ GPa, $G = 80$ GPa, $\sigma_y = 250$ MPa; every number recomputed in the course suite.
- Units 6.1-6.2 load along and about the axis; 6.3-6.4 across; 6.5 rotates the state; 6.6 asks when it stops being a member.

Axial loading: stress, strain, held cold

the tie rod: one dimension, four traps

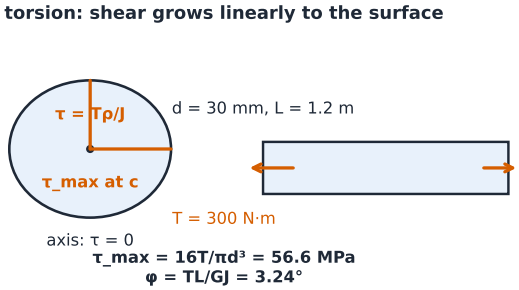
held cold by 40 K: σ_T = EαΔT = 96.0 MPa — no force needed



σ = P/A = 221.0 MPa · δ = PL/AE = 1.658 mm · ε = 0.001105

Tie rod $d = 12$ mm, $P = 25$ kN: $\sigma = P/A = \mathbf{221.0}$ MPa, $\delta = PL/AE = \mathbf{1.658}$ mm. Held cold by 40 K: $\sigma_T = E\alpha\Delta T = \mathbf{96.0}$ MPa with no force.

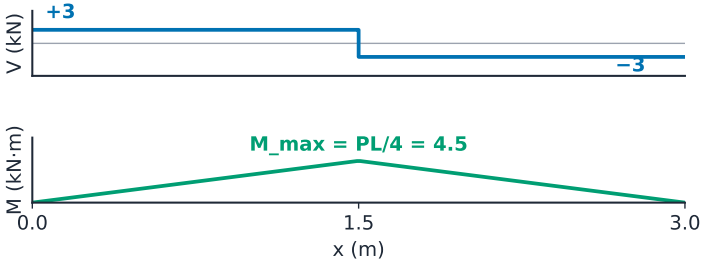
Torsion: shear to the surface, twist along L



Shaft $d = 30$ mm, $T = 300$ N $\cdot$ m: $\tau_{max} = 16T/\pi d^3 = \mathbf{56.6}$ MPa, $\phi = TL/GJ = \mathbf{3.24}^\circ$. At 180 rpm it sells $P = T\omega = \mathbf{5.65}$ kW.

Shear and bending: the diagrams

central load: P = 6 kN on a 3 m span, reactions 3 kN each

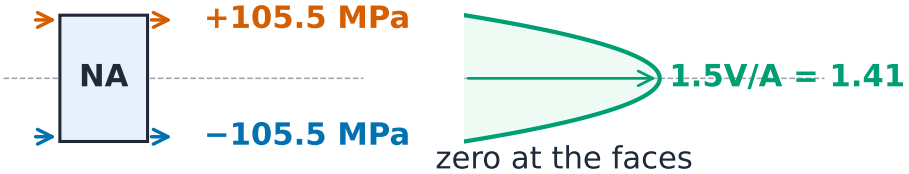


Central $P = 6$ kN on 3 m: reactions 3 kN, V steps $+3 \to -3$ kN, $M_{max} = PL/4 = \mathbf{4.5}$ kN · m. Uniform $w = 2$ kN/m: $M_{max} = wL^2/8 = \mathbf{2.25}$ kN · m.

Beam stresses: linear and parabolic

bending is linear in depth, shear is parabolic

40 × 80 mm at M = 4.5 kN·m

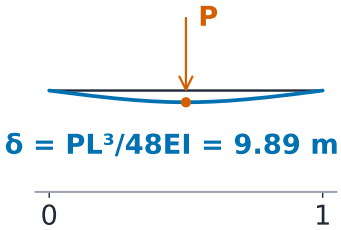


On the 40×80 mm arm: $\sigma = Mc/I = \mathbf{105.5}$ MPa (or M/S with $S = \mathbf{42.7} \times 10^{-6}$ m^3), and $\tau_{max} = 1.5V/A = \mathbf{1.41}$ MPa on the neutral axis.

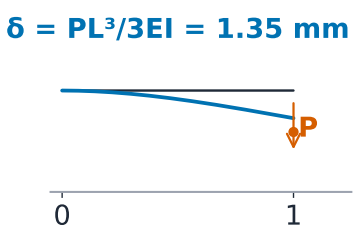
Deflection: stiffness is a spec

two standard cases, two cubes of L

mid-span, central load

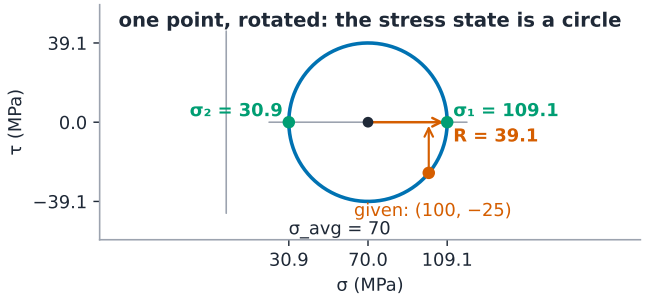


cantilever, tip load



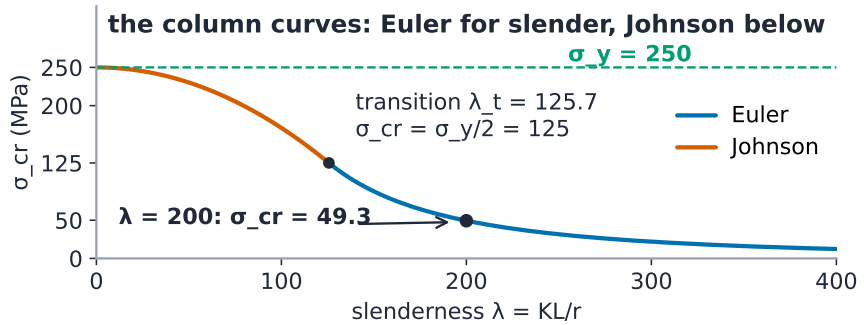
Central 6 kN on 3 m: $\delta = PL^3/48EI = \mathbf{9.89}$ mm. Cantilever 1.2 m with 800 N: $\delta = PL^3/3EI = \mathbf{1.35}$ mm - double the depth, eighth the sag.

Combined stress and Mohr's circle



Point (100, 40, 25) MPa: centre 70, $R = \mathbf{39.1}$, so $\sigma_1 = \mathbf{109.1}$, $\sigma_2 = \mathbf{30.9}$ MPa at $\mathbf{19.9}°$. With the second stress at -40: $R = \mathbf{85.4}$, $\sigma_1 = \mathbf{125.4}$, $\sigma_2 = \mathbf{-45.4}$ MPa.

Buckling and failure criteria



Rod $d = 20$ mm, 1 m pinned: $P_{cr} = \mathbf{15.5}$ kN at $\sigma_{cr} = \mathbf{49.3}$ MPa, $\lambda = 200$; Johnson takes over below $\lambda_t = \mathbf{125.7}$. At $n = 2.5$: $P_{allow} = \mathbf{6.2}$ kN.

The bracket file, one method

1. **Along the axis** $\sigma = 221.0$ MPa, $\delta = 1.658$ mm, held-cold 96.0 MPa; shaft: $\tau = 56.6$ MPa, $\phi = 3.24°$, 5.65 kW.
2. **Across the section** $M_{max} = 4.5$ kN $\cdot$ m, $\sigma = 105.5$ MPa; $\tau_{max} = 1.41$ MPa; Mohr: $R = 39.1$, $\sigma_1 = 109.1$ MPa, $\sigma' = 97.4$ MPa.
3. **Move and stay** $\delta = 9.89$ mm mid-span, 1.35 mm at a tip; strut $P_{cr} = 15.5$ kN at $\lambda = 200$, $P_{allow} = 6.2$ kN at $n = 2.5$.

Check yourself

- The rod and the shaft? (221.0 MPa, 1.658 mm; 56.6 MPa, 3.24°, 5.65 kW.)
- The diagrams and the stresses? ($PL/4 = 4.5$ and $wL^2/8 = 2.25$ kN · m; $\sigma = 105.5$, $\tau = 1.41$ MPa.)
- The sag, the circle, the verdict? (9.89 mm; $R = 39.1$, $\sigma_1 = 109.1$, $\sigma' = 97.4$ MPa.)
- The strut and its factor? ($P_{cr} = 15.5$ kN at $\lambda = 200$; $P_{allow} = 6.2$ kN at $n = 2.5$.)