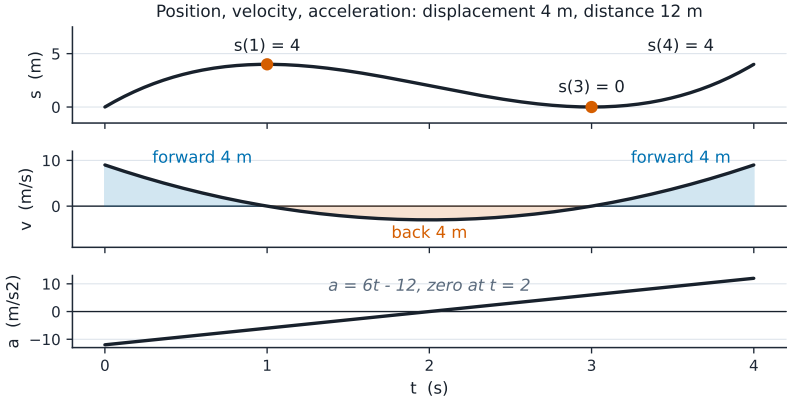
Mathematical Methods for ME

Curated undergraduate study track (not accredited, no external exam). Original explanations and practice only, following the reviewed course handout.

Six instruments, one rig

- The vibration test rig: every unit is an instrument applied to it - calculus reads the trace, algebra conditions the solve, the ODE is the rig itself, numerics run what has no closed form, fields carry the heat, statistics report the shaft.
- The habit - **every number recomputes**, and each is checked a second way before it is trusted.

Calculus: the trace is a dictionary

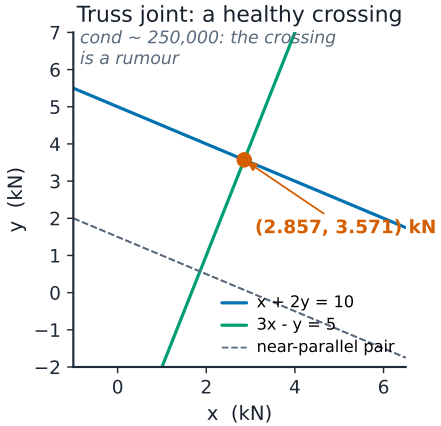


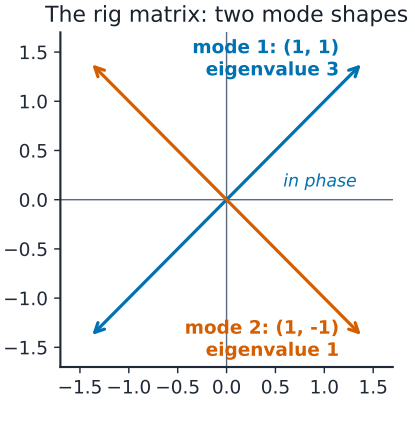
Displacement 4 m but distance **12 m**: v changes sign at t = 1 and 3.

Calculus in action: the junior engineer's 4 m

1. **Find** v < 0 between t $= 1$ and 3 - the trolley retraces.
2. **Compute Displacement** 4 m and distance 12 m differ, so the motion reversed - count both hatches.
3. **Sign** Zeroes of v before the signature; disks and quadrature to 4 decimals for the fillet.

Linear algebra: the condition number prices trust



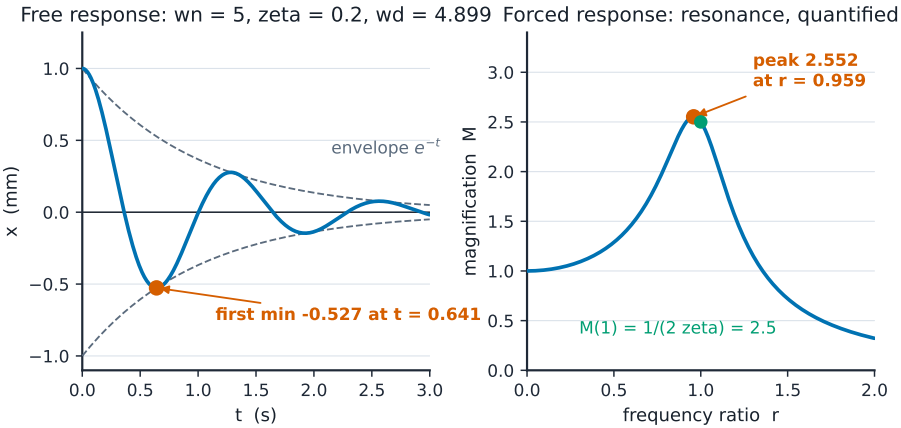


The truss lines cross at **(2.857, 3.571)** kN; the rig modes are (1, 1) and (1, −1).

Linear algebra in action: spread the calibration points

1. **Condition** Two close calibration points are nearly parallel lines - the slope wobbles with the noise.
2. **Fix** Widen the mass range; κ falls. No new physics bought.
3. **Split** Any free motion is half in-phase, half opposed - each part oscillates at its own $\sqrt{(k/m)}$.

Differential equations: the rig is the equation



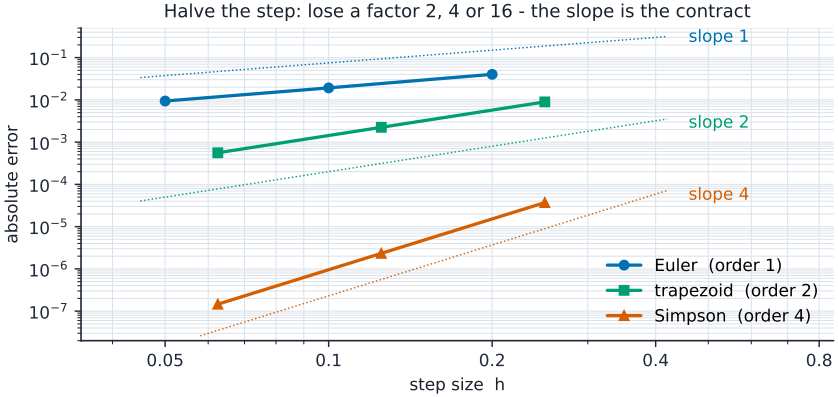
$\zeta = 0.2$: first minimum **−0.527** at t = 0.641; **resonance** peak **2.552** at r =

exercises.ipl exercises.com.cn

Differential equations in action: the floor fails the mount

1. **Read** $r = 4.79/5 = 0.959$ - the peak. $M = 2.552$; with the moving base 2.73, so 1 mm of floor becomes about 2.7 mm at the till.
2. **Fail** The mount's clearance is 2 mm - it fails, quantitatively.
3. **Fix** Stiffen (r down, left slope), damp (peak falls as $1/2\zeta$), or isolate (right slope - soft mounts under sensitive gear).

Numerical methods: the slope is the contract



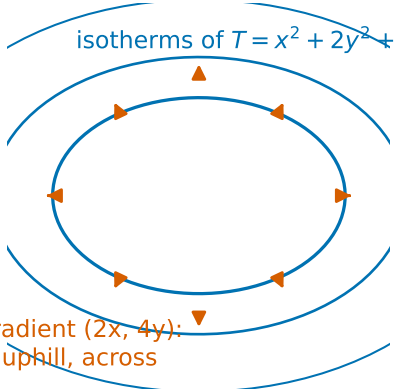
Halve the step: Euler error ÷2, trapezoid ÷4, Simpson ÷16 - the slope is the

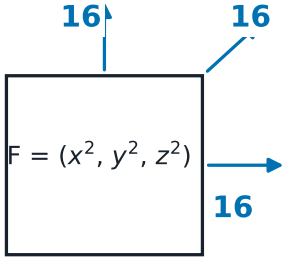
Numerical methods in action: h from the budget

1. **Budget** Error $\approx 0.19h$ for Euler (the measured rate), so h $\approx$ 1e−4 - ten thousand steps.
2. **Escalate** The RK-class stepper this course builds toward runs h ≈ 0.1 - ten steps, two hundred times fewer, same answer.
3. **Floor** Push Simpson to a million panels and round-off (~1e−10) dominates - the measured flattening IS the floor.

Vector calculus: count what leaves

Fields: T rises at 7.483 K/m; q = -40 grad T Flux 48, counted two ways





surface flux: 16 + 16 + 16 = 48

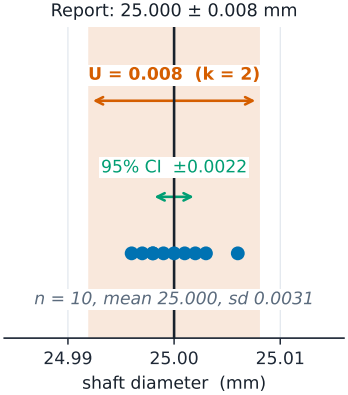
divergence: 2(x+y+z) dV = 48

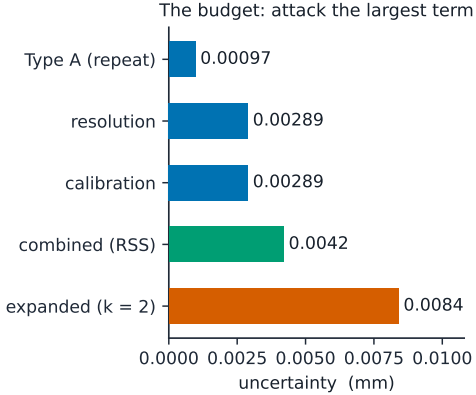
∇T gives **7.483** K/m; **flux 48** by surface = **48** by **divergence**.

Vector calculus in action: the 96 mW amplifier

1. **Field** $-k\nabla^2 T = -480\ \mathrm{W/m^3}$ over the 0.0002 $\mathrm{m^3}$ volume - the block ABSORBS 96 mW.
2. **Check** The datasheet says it dissipates 100 mW. The size matches, the sign does not - a bowl-shaped T is a cooled region, so this fit is not the amplifier's interior.
3. **Repair** Open surface? Patch the hole with a convenient surface and subtract - how the method of images feels from inside.

Probability: the interval travels with the number





Report **25.000 ± 0.008** mm (k = 2): resolution and calibration dominate.

Probability in action: ninety more readings?

1. **Read** Resolution and calibration (0.00289 each) dominate the statistics (0.00097) - more readings move the fourth decimal.
2. **Attack** The largest term: a 0.001-resolution micrometer halves it; patience does not.
3. **Carry** Stiffness k $=$ mg$/\delta$ with $\delta = 2.50$ mm: the shaft's 0.008 mm is 0.32%, over the mass's 0.1% - report k(1 ± 0.0034).

The project sheet, reproducible

1. **Motion Displacement** 4 m, distance 12 m; fillet volume $\pi \times 17.067 = 53.62$ m^3.
2. **Algebra** Truss joint (2.857, 3.571) kN; eigenvalues 3, 1 with modes (1, 1), (1, -1); condition numbers 3 and 250,008.
3. **Engine** ω_n $= 5$, $\zeta = 0.2$, ω_d $= 4.899$; first minimum -0.527; M(1) $= 2.5$, peak 2.552 at r $= 0.959$. Euler $\div 2$, trapezoid $\div 4$, Simpson $\div 15.9$. **Flux** $48 = 48$; circulation 2π; ∇^2T $= 12$. u_c $= 0.0042$, U $= 0.008$, A $= 490.9 \pm 0.3$ mm^2.

Check yourself

- The two motion numbers and the fillet? (**Displacement** 4 m, distance 12 m; 53.62 m^3.)
- The joint, the modes and the rumour? ((2.857, 3.571) kN; 3 and 1 with (1, 1), (1, -1); $\kappa \approx 250{,}000$.)
- The rig's ODE numbers? (ω_d $= 4.899$; -0.527 at t $= 0.641$; peak 2.552 at r $= 0.959$.)
- The rates, the **flux** and the interval? ($\div 2$, $\div 4$, $\div 15.9$; $48 = 48$; 25.000 ± 0.008 mm $\to$ 490.9 ± 0.3 mm^2.)