Control Systems

Curated undergraduate study track (not accredited, no external exam). Original explanations and practice only, following the reviewed course handout.

Closing the loop

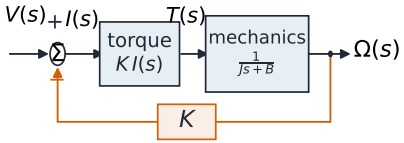
- EE06 wrote the open-loop system function; this course closes the loop and asks how to make the output follow the input: fast, accurate, without ringing.
- Every design is checked in both domains: a margin predicts a damping, a pole placement predicts an **overshoot**, and Routh, locus and Nyquist must agree.

Modelling: two laws, one loop

1. **RC** R = 2 kOhm, C = 100 uF: tau = 0.2 s; a 5 V step reaches 3.16 V (63.2 percent) at one **time constant**.
2. **Motor** J = 0.01, B = 0.1, K = 0.5, R = 1: omega/V = K over (Js+B)R + K squared = 50/(s+35); tau = 28.6 ms, 1.43 rad/s per volt.

The motor as it is wired

torque law $T = K i$ Newton $J\dot{\omega} = T - B\omega$



KVL: $V = R i + K\omega$

with $L \approx 0$: $\dfrac{\Omega(s)}{V(s)} = \dfrac{K}{(Js+B)R + K^2} = \dfrac{50}{s+35}$

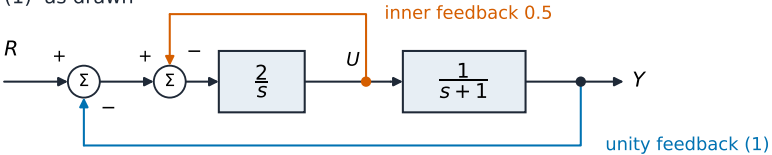
Torque law and Newton close through the **back-EMF** into one inner loop.

Transfer functions and block diagrams

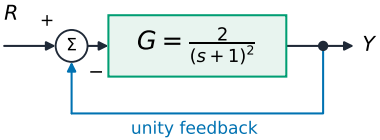
1. **One loop** G1 = 1/(s+1), G2 = 1/(s+2), K = 6, unity **feedback**: T = 6/(s squared + 3s + 8); omega n = 2.83, zeta = 0.530.
2. **Nested** Inner 2/s with 0.5 collapses to 2/(s+1); series with 1/(s+1); outer unity gives 2/(s squared + 2s + 3); zeta = 0.577.

Three reductions, one picture

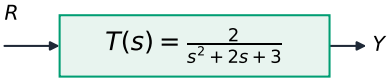
(1) as drawn



(2) inner loop collapsed, series multiplied



(3) close the loop: $T = G/(1+G)$



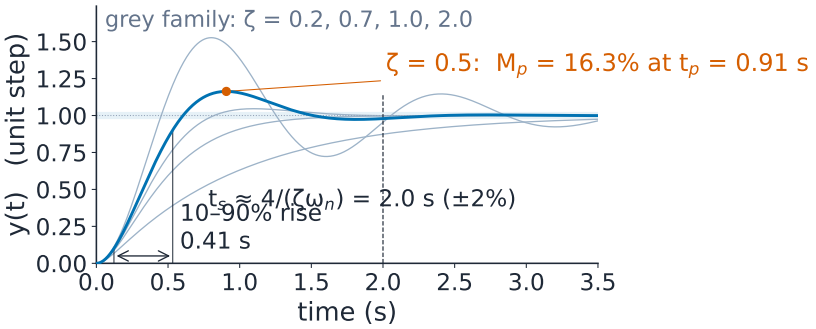
$\omega_n = \sqrt{3} = 1.73$ rad/s, $\zeta = 0.577$

Inner loop first, series second, outer loop last: each panel is one rule.

Time-domain response: the second-order dictionary

1. **Specs** omega n $= 4$, zeta $= 0.5$: **overshoot** 16.3 percent, peak 0.907 s, settling 2.0 s (2 percent band), rise 0.41 s.
2. **Design** K over s(s+2) with zeta $= 0.45$: K $= 4.94$, K_v $= 2.47$, ramp error 0.405, step error exactly zero.

One damping ratio, one shape

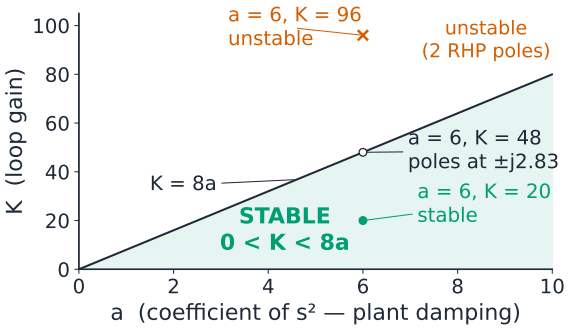


The $\zeta = 0.5$ curve annotated with **overshoot**, peak, band and rise.

Routh: count, do not solve

1. **Quartic** s4 + 2s3 + 3s2 + 4s + 5: first column 1, 2, 1, minus 6, 5: two sign changes, two RHP roots (exact: plus 0.29 plus-minus j1.42).
2. **Range** s3 + 5s2 + 6s + K: the s1 row is (30 minus K)/5, so $0 < K < 30$; at $K = 30$ the auxiliary gives plus-minus j2.45.

The stable design space

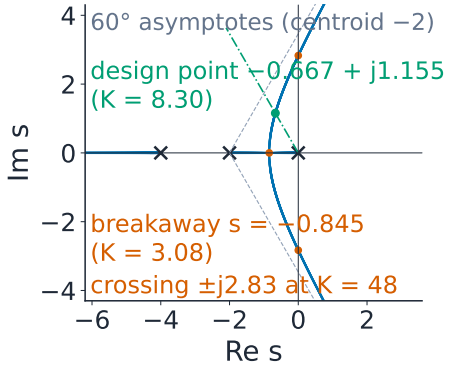


For $s^3 + a\,s^2 + 8\,s + K$, stability is exactly $0 < K < 8a$, a straight boundary.

The locus of one knob

1. **Sketch** K over s(s+2)(s+4): centroid minus 2, asymptotes 60 degrees, breakaway minus 0.845 at K = 3.08, crossing plus-minus j2.83 at K = 48.
2. **Design** zeta = 0.5 ray meets the locus at minus 0.667 + j1.155 with K = 8.30; the third pole minus 4.67 stays well left.

Poles leave, meet and bend



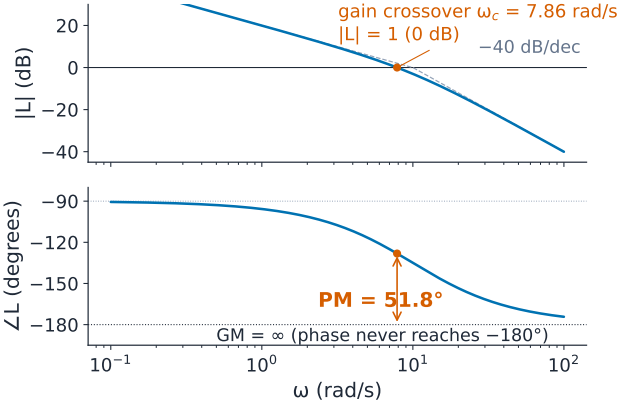
Breakaway at minus 0.845, asymptotes from minus 2, jw crossing at plus-minus j2.83 ($K = 48$).

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Margins: the engineer's tolerance

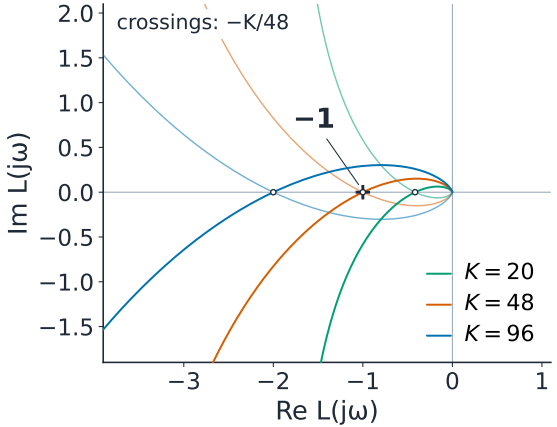
1. **PM by arithmetic** 100 over s(s+10): crossover 7.86 rad/s, phase minus 128.2 degrees, PM = 51.8 degrees, GM infinite; exact closed loop zeta = 0.5.
2. **Three ways** 20 over s(s+2)(s+4): GM = 2.4 (7.6 dB), PM = 25.7 degrees at 1.73 rad/s; Nyquist wraps twice at K = 96: Z = 2, Routh agrees.

Bode and Nyquist analysis: straight lines with a verdict



Read the **phase margin** at **gain crossover**; the phase never reaches −180°, so

One shape slides through minus one



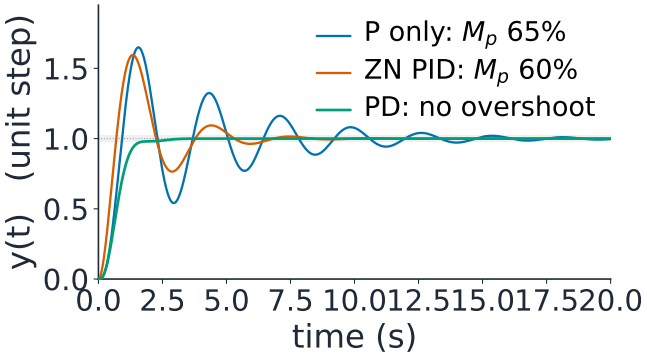
Gain scales the curve; K = 48 passes through minus one, K = 96 wraps twice.

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PID: three terms, three prices

1. **ZN** Ku = 48, Tu = 2.22 s: Kp = 28.8, Ti = 1.11 s, Td = 0.278 s; P-only rings 65 percent, the ZN PID rings 60 percent and removes the ramp error.
2. **Retune** Same Kp with Td = 0.5 s: poles minus 2 and minus 2 plus-minus j3.22, no **overshoot**, settling 2.1 s (**derivative** on the measurement; on the error it rings 14 percent); the ramp error stays 1/K_v = 8/Kp = 0.278.

One plant, three tunings

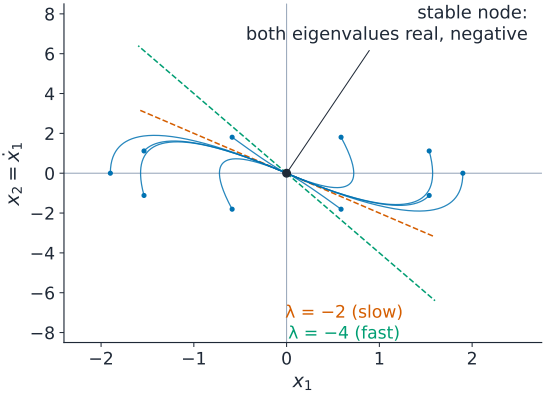


P only overshoots 65 percent; Ziegler-Nichols rings at 60 with zero ramp error; the PD retune (**derivative** on the measurement) shows none.

State space: poles by algebra

1. **Decay** 8 over (s+2)(s+4): Phi(0.5) has entries 0.600, 0.116, minus 0.930, minus 0.097 and eigenvalues e to the minus 1 and e to the minus 2.
2. **Place** Both poles at minus 10: matching s2 + 20s + 100 gives Kfb = [92, 14]; the eigenvalues of A minus BK land exactly on minus 10.

Trajectories straighten onto the slow pole



A stable node: every trajectory bends onto the lambda = minus 2 eigenvector.

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Check yourself

- Derive the motor model from its two laws and name its **time constant** and DC gain.
- Reduce a nested **block diagram** to one fraction and read the **natural frequency** and damping from the denominator.
- Count RHP roots with the array, price a damping ray in gain, and read both margins off a Bode sketch.
- Tune the three PID terms against their price list, and place both poles of a two-state model by algebra.