Signals and Systems

Curated undergraduate study track (not accredited, no external exam). Original explanations and practice only, following the reviewed course handout.

What does a system do to ANY input

- EE04 asked what one circuit does at one frequency; this course answers for any input.
- Convolution, Fourier, and z - the three answers the rest of the degree runs on.

System properties: test from the definition

1. **Trap** $y = x(2t)$ is time-VARYING: a shifted input gives a half shift ($u(2t\text{-}1) = u(t\text{-}1/2)$), not the full one.
2. **Test** Compare shift-then-system against system-then-shift - the one-line test every property question reduces to.
3. **Stability** BIBO for LTI: integral of $|h|$ finite - the integrator fails, the exponential passes.

Four tests, four verdicts

linearity

feed x1 + x2
output = y1 + y2?

sum in, sum out

time invariance

shift x by T
output shifts by T only?

shift in, shift out

causality

output at t = 0
depends on x(t > 0)?

no output before input

BIBO stability

bounded x in
bounded y out?

integral |h| < infinity

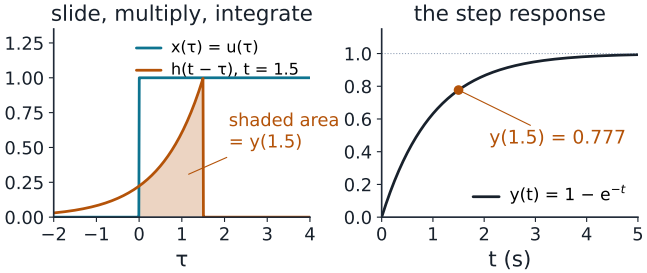
Linearity, **time invariance**, **causality**, stability: each property is one probe test.

exercises.iol exercises.com.cn

Convolution: superposition made mechanical

1. **Step** u * e^-t u = 1 - e^-t: the first-order step response, both endpoints check (0 at 0, 1 at infinity).
2. **Widths** Two width-2 rectangles convolve to a width-4 triangle peaking at 2.
3. **Area** e^-2t * e^-3t = e^-2t - e^-3t; its integral 1/2 - 1/3 = 1/6 equals the product of the input areas.

Slide, multiply, integrate

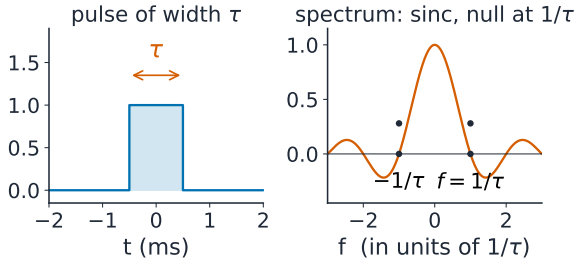


The flipped h slides past x; each output value is the overlap area - y(1.5) = 0.777 for the step into e^-t.

Fourier: convolution becomes multiplication

1. **Pair** Width-tau rectangle transforms to tau sinc(omega tau/2): value at 0 is the area, first null at f $= 1/$tau.
2. **Shift** x(t-3) maps to X e^{-j3 omega}; modulation moves the spectrum to the carrier.
3. **Response** H $= 1/(1+$j omega) at omega 1: gain 1/sqrt2, phase -45 degrees - the EE05 corner, found by transforms.

Pulse to sinc, null at 1/tau

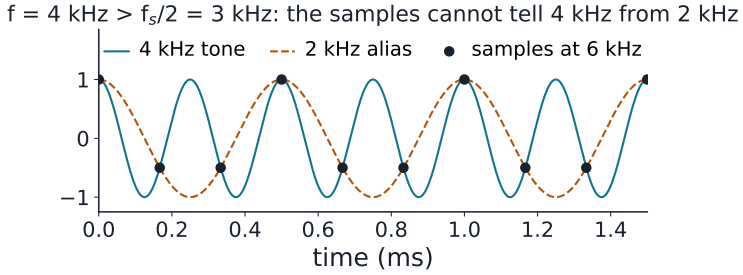


A rectangle in time transforms to a sinc in frequency; the 1 ms click spans to 1 kHz, the 0.1 ms click to 10 kHz.

Sampling: replicas, Nyquist, and the fold

1. **Alias** 4 kHz at 6 kHz sampling: $f_s/2 = 3$ kHz, so the tone reads as $6 - 4 = 2$ kHz.
2. **Verify** $\cos(2\pi\, 4n/6)$ and $\cos(2\pi\, 2n/6)$ agree sample by sample - the even-cos identity.
3. **Fix** 44.1 kHz audio with 38 kHz interference: the alias lands at 6.1 kHz; filter before sampling or oversample.

Two tones, one sequence

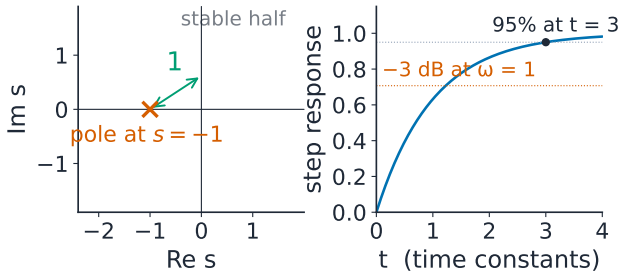


Sampled at 6 kHz, a 4 kHz cosine and a 2 kHz cosine pass through exactly the same samples - the fold is complete.

System functions: poles tell the story

1. **One pole** 1/(s+1): h = e^-t u; step 1 - e^-t; at j omega 1, gain 1/sqrt2 - three views, one system.
2. **Resonance** s/(s^2+2s+101): poles -1 +- j10, a band-pass peaking near omega 10 with gain about 0.5.
3. **Feedback** K/((s+1)(s+2)) closed loop: second-order stability for K $>$ -2; at K=2 the poles sit at -1.5 +- j1.32.

One pole, three readings

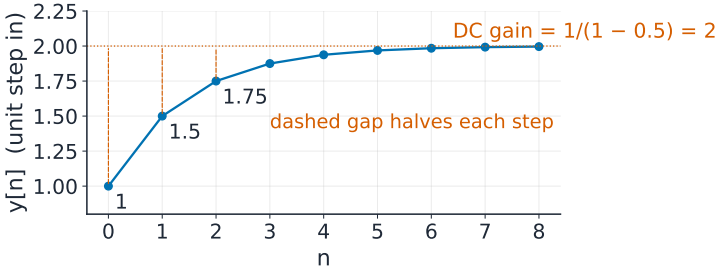


The pole at s = -1 is the decay e^-t, the -3 dB point at omega = 1, and the 95% settle at t = 3 - one number, three readings.

Discrete systems: roots inside the unit circle

1. **Accumulator** y = 0.5y + x: step response 1, 1.5, 1.75, 1.875 - converging to the DC gain 2.
2. **Oscillator** y = -0.25y[n-2] + x: roots +-j0.5, a period-4 mode under a 0.5^n envelope; DC gain 0.8.
3. **Averager** 3-point mean: DC gain 1, alternating gain 1/3 - attenuation exactly as the sum predicts.

The gap halves each step

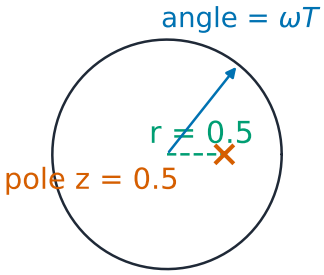


y[n] = 0.5 y[n-1] + x[n] on a step: 1, 1.5, 1.75, 1.875 closing on the DC gain 2 - the table IS the proof.

The z-transform: Laplace wrapped around

1. **Accumulate** 1/(1 - 0.5 z^-1): step response 2(1 - 0.5^{n+1}) - matches the iteration at n = 0 and at infinity.
2. **Differencer** (1 + z^-1)/(1 - 0.5z^-1) on a step: y[n] = 4 - 3(0.5)^n; iteration confirms 1 then 2.5.
3. **High-pass** (1 - z^-1)/2: zero at DC, one at Nyquist, sin(omega/2) in between - the anti-averager.

Frequency as angle, damping as radius



the circle is the frequency axis; radius 0.5 = damping -693 s^{-1} at T = 1 ms

> A pole at z = 0.5 with T = 1 ms is a decay of -693 per second.

Check yourself

- Properties: is $y = x(2t)$ time-invariant? (No - half shift.)
- Sampling: 4 kHz at 6 kHz reads as? (2 kHz.)
- z-plane: causal system, pole at 0.5 means? (Stable - inside the **unit circle**.)