Mathematical Methods for EE

Curated undergraduate study track (not accredited, no external exam). Original explanations and practice only, following the reviewed course handout.

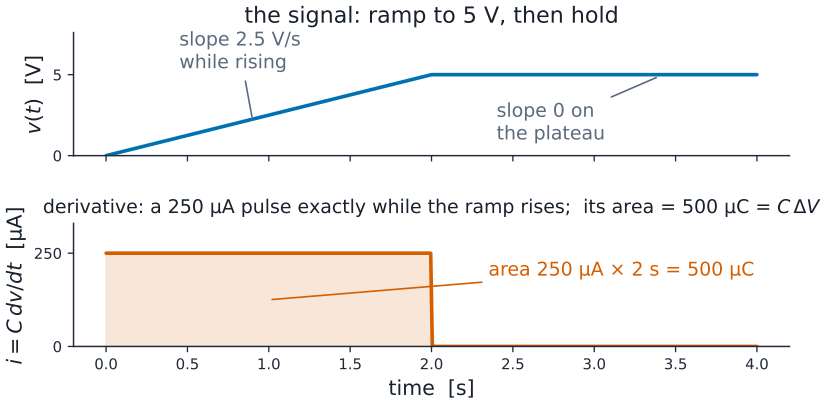
The toolkit the degree runs on

- Algebra refresh, then dynamics tools, then randomness and fields.
- Check every symbolic answer numerically at a concrete value.

Calculus for signals: slope, area, and the capacitor identity

1. **Current** $v = 2.5t$, $C = 100\ \mu F$: $i = C\, dv/dt = 250\ \mu A$ constant.
2. **Charge** $\int i\, dt = 500\ \mu C$.
3. **Check** $Q = C \cdot \Delta V = 100\mu \times 5 = 500\ \mu C$ — different method, same answer.

Slope and area are the two readings

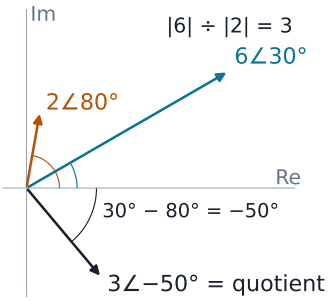


A 2.5 V/s ramp draws 250 µA into 100 µF; the pulse area 500 µC equals C ΔV.

Complex numbers: polar arithmetic is the EE route

1. **Divide** $6\angle 30^\circ \,/\, 2\angle 80^\circ = 3\angle -50^\circ$ — read off.
2. **Check** $3\angle -50^\circ \times 2\angle 80^\circ = 6\angle 30^\circ$ back to the start.
3. **Conjugate** $|z|^2 = z \cdot z^*$ turns division into multiplication: $1/(3+j4) = (3-j4)/25$.

Moduli divide, arguments subtract

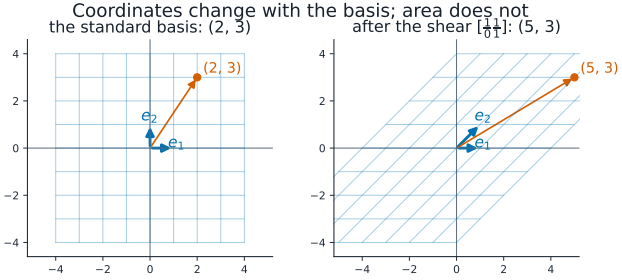


6 at 30 degrees over 2 at 80 degrees is 3 at -50 degrees directly; the round trip back home is the check.

Linear algebra: the determinant is a circuit-debugging tool

1. **Solve** $x + 2y = 8$, $3x + y = 9$: eliminate $\rightarrow (2, 3)$.
2. **Verify** $\det = -5$; the inverse returns $(2, 3)$ again.
3. **Read** Nodal analysis IS $Ax = b$ — **singular** A flags the modelling error.

A basis spans; a matrix moves you

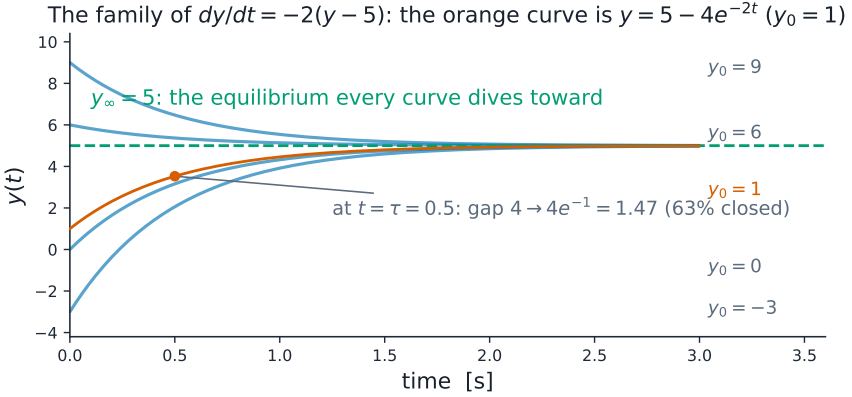


The point (2,3) in the standard basis becomes (5,3) under the shear: coordinates change, geometry does not.

Ordinary differential equations: two routes, one answer

1. **Classical** $dy/dt + 2y = 10$, $y(0)=1$: $y = 5 + C\exp(-2t)$, $C = -4$.
2. **Laplace** $Y = 5/s - 4/(s+2)$ — inverts to the same y.
3. **Poles** $s = 0$ (constant), $s = -2$ (decay): the shape before inversion.

One equation, a family of curves



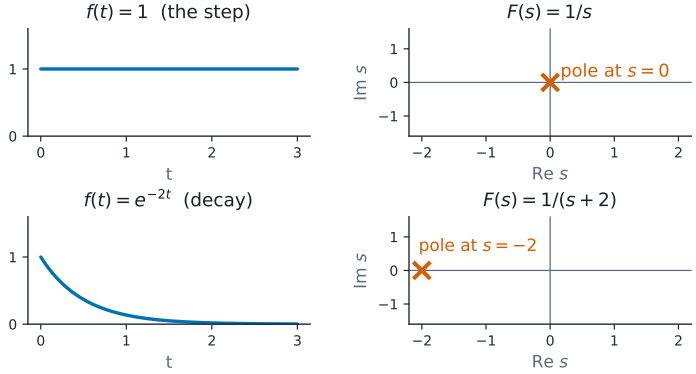
Every curve dives toward y = 5; the **initial condition** selects y = 5 − 4e^(−2t).

Laplace transforms: the ODE becomes algebra

1. **Transform** $dy/dt + 2y = 10$, $y(0)=1$: $sY - 1 + 2Y = 10/s$.
2. **Split** $Y = 5/s - 4/(s+2)$.
3. **Invert** $y = 5 - 4\exp(-2t)$ — identical to the classical route, by construction.

Each curve maps to a pole

Each time curve maps to a pole; the pole position predicts the decay



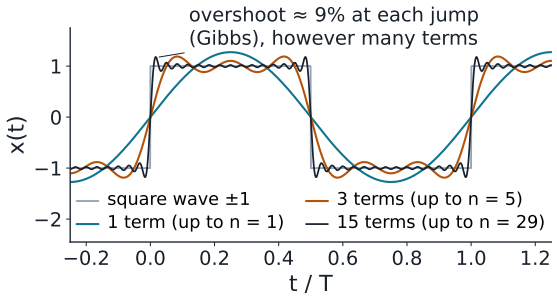
The step's pole sits at s = 0, the decay's at s = −2: pole position predicts be-

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Fourier series: sharp edges need many harmonics

1. **Square wave harmonic** b $= 4/(n\pi)$, odd n only: 1.27, 0.42, 0.25...
2. **Check** At the peak, $4/\pi(1 - 1/3 + 1/5 - ...) = 1$ exactly — the Gregory series.
3. **Truncate** Keep two harmonics: 0.849 at the peak — 15% low, ripple at edges.

Odd harmonics build the square

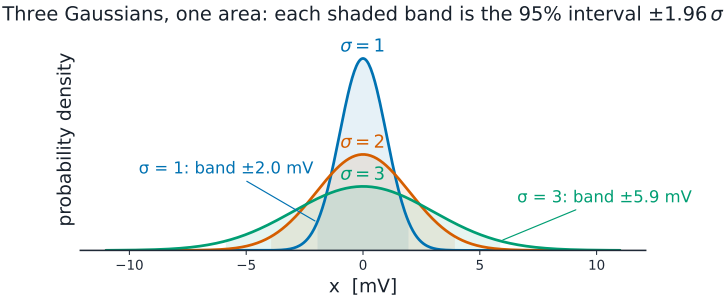


Partial sums of 1, 3 and 15 harmonics hug the square wave; the 9% Gibbs overshoot never leaves.

Probability: variances add, and averaging divides by $\sqrt{n}$

1. **Combine** σ_1=2, σ_2=3 mV: combined $\sigma = \sqrt{13} = 3.61$ mV, not 5.
2. **Average** 16 averages of 2 mV noise: $\sigma/\sqrt{16} = 0.5$ mV.
3. **Rationale** $\sqrt{n}$ is why instrument averaging is the cheapest noise filter.

Same area, different spread

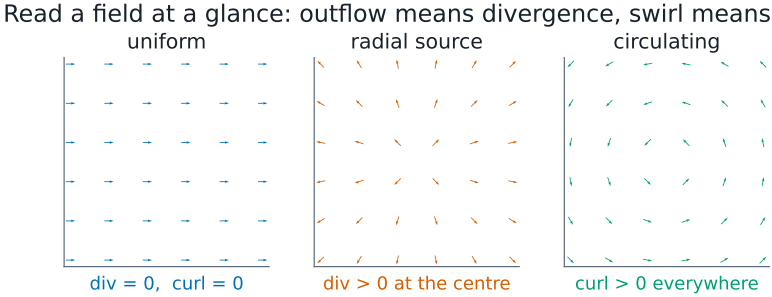


Three Gaussians with sigma 1, 2, 3: wider means shorter, the 95% band is plus-minus 1.96 sigma.

Vector calculus: three operators, verified by a small step

1. **Field** $V = 3x^2y$: $\nabla V = (6xy,\ 3x^2) = (12,\ 3)$ at $(1,\ 2)$; $E = -(12,\ 3)$ V/m.
2. **Check** Step $\delta x = 0.01$: $\Delta V \approx 0.12$ V, and $-E \cdot \delta = 0.12$ V ✓.
3. **Theorems Divergence** and Stokes theorems become Gauss and Faraday laws.

Outflow means divergence, swirl means curl



Uniform flow sources nothing and swirls nothing; the radial field diverges at the centre; the circulating field curls.

Check yourself

- **Phasors**: $(8\angle 15°)/(2\angle 40°)$? $(4\angle -25°.)$
- Laplace: **poles** at 0 and -2 give what shape? (Constant plus decay.)
- Noise: combine $\sigma = 2$ and 3 mV? ($\sqrt{13} \approx 3.61$ mV.)