Circuit Analysis I (DC)

>- Curated undergraduate study track (not accredited, no external exam). Original explanations and practice only, following the reviewed course handout.

A ladder of methods, each rung used by the next

- Units and sign conventions first; systematic methods second; energy storage last.
- Every worked example is checked a second way - a different **loop**, node or limit.
- The checking habit is the course's real content.

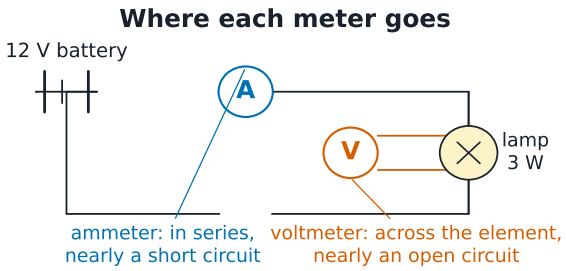
Six quantities form one chain

- **Charge** moves; its rate is current; current across a potential difference transfers energy.
- **Power** is energy per time; 1 kW · h = 3.6 MJ.
- Prefixes carry the arithmetic - convert to SI units first.

The sign convention decides supply and demand

1. **Draw** 12 V battery, 0.25 A lamp, arrow into the lamp's + terminal.
2. **Compute** $p = vi = 12 \times 0.25 = 3$ W absorbed by the lamp; the battery absorbs -3 W.
3. **Close** Energy over 2 min: $w = pt = 360$ J. Supply matches demand exactly.

Measurement joins the circuit, carefully



The **ammeter** must carry the load current (**series**, nearly a short); the voltmeter must not (across, nearly an open).

Ohm's law, and combining resistors honestly

1. **Network** 9 V source, 6 kΩ in **series** with 3 kΩ.
2. **Current** $i = 9\ \text{V} / 9\ \text{k}\Omega = 1\ \text{mA}$
3. **Share** $v_2 = 1\ \text{mA} \times 3\ \text{k}\Omega = 3\ \text{V}$; check $-9 + 6 + 3 = 0$ around the **loop**.

Kirchhoff's laws are conservation, twice

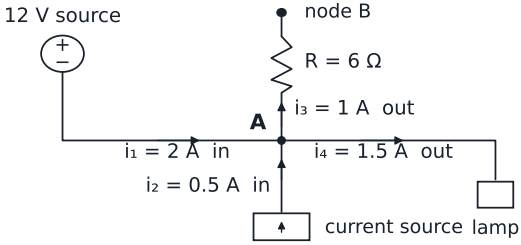
1. **Node A** 2 A and 0.5 A in; 1 A out through a third branch.
2. **Signs** Out positive: $-2 - 0.5 + 1 + \mathrm{i}_4 = 0$
3. **Answer** $\mathrm{i}_4 = 1.5$ A leaves. A negative answer is information, not a mistake.

Reference directions make KCL mechanical

One node, four signed branch currents

KCL at A (signed sum out = 0): $-2 - 0.5 + 1 + 1.5 = 0$

Choose every arrow before computing; a negative answer is information, not a mistake.



Two currents in, two out - the signed sum at node A closes to zero.

Nodal analysis - KCL does the walking

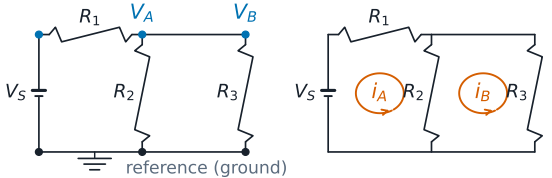
1. **Set up** 12 V through 2 Ω to A; 6 Ω to ground; 1 A into A: $(V_A-12)/2 + V_A/6 - 1 = 0$
2. **Solve** $3(V_A-12) + V_A - 6 = 0$ gives $V_A = 10.5$ V
3. **Check** Source branch 0.75 A + 1 A in; $10.5/6 = 1.75$ A out - balanced.

Mesh analysis - KVL around each window

1. **Loops** 12 V with 1 Ω, sharing 2 Ω with a source-free 2 Ω **loop**.
2. **Equations** $3I_1 - 2I_2 = 12$ and $I_1 = 2I_2$
3. **Solve** $I_1 = 6$ A, $I_2 = 3$ A; loop 1 check $-12 + 6 + 6 = 0$.

Nodal labels or mesh loops: same circuit, two labellings

Nodal: node voltages **Mesh: loop currents**



one KCL equation per unknown node shared branch carries $i_A - i_B$

Left: unknown node voltages above a grounded reference. Right: circulating mesh currents whose difference crosses the shared branch.

Network theorems: Thévenin reduces the whole network

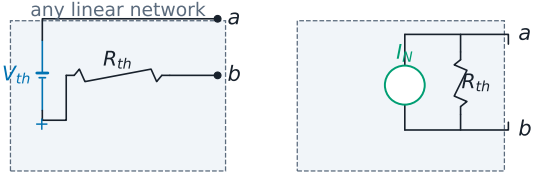
1. **Network** 10 V with 4 Ω **series** feeding a 6 Ω shunt; load across the shunt.
2. **Reduce** V_th $= 10 \times 6/10 = 6$ V; R_th $= 4 \parallel 6 = 2.4$ Ω
3. **Use** Any load now faces one 6 V source and one 2.4 Ω **resistor**.

Maximum power arrives at 50% efficiency

- Match R_L = R_th for maximum load **power**, P_max = V_th^2/(4R_th).
- At the match, R_th burns exactly what the load receives.
- **Superposition** adds responses, never powers.

Every linear one-port is one source and one resistance

Thévenin: series form **Norton: parallel form**



measure: V_{th} open-circuit, R_{th} with sources zeroed

swap the pair freely: $I_N = V_{th}/R_{th}$

Measure the **open-circuit voltage** and the zeroed-source resistance; the whole network folds into Vth in **series** with Rth, or IN in **parallel**.

Two golden rules solve every ideal op-amp circuit

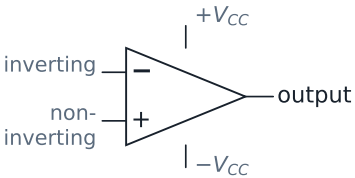
1. **Circuit** R_in $= 10$ kΩ to the $-$ node, R_f $= 100$ kΩ feedback, $+$ grounded.
2. **Current** v_in/R_in flows entirely through R_f (no input current).
3. **Gain** v_out/v_in $= -$R_f/R_in $= -10$; 0.2 V in gives -2 V.

Saturation ends the linear story

- The output cannot leave the supply rails; a ± 15 V op-amp stops near ± 14 V.
- Check $|\text{gain} \times \text{input}|$ against the rails before quoting any output.
- Beyond **saturation** the **virtual short** is simply false.

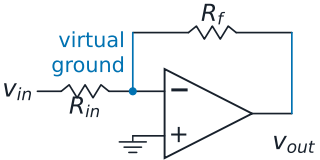
Five terminals and one taming trick

Five terminals



ideal: infinite gain,
infinite input resistance

Inverting amplifier



the − node sits at 0 V, so
all of v_{in}/R_{in} flows through R_f

The triangle has two inputs, one output and two supply rails; **negative feedback** pins the minus node to the plus node - the **virtual ground** that makes gain exact.

Capacitors and inductors store, resistors burn

1. **Capacitor** 100 µF at 12 V holds $\frac{1}{2} \times 100\mu \times 144 = 7.2$ mJ
2. **Inductor** 20 mH at 3 A holds $\frac{1}{2} \times 0.02 \times 9 = 90$ mJ
3. **Steady state** DC leaves the **capacitor** open and the **inductor** short - most problems end here.

Burn, or store in a field

Burn it, or store it in a field

Resistor



$v = Ri$

burns energy

$p = i^2R$

stores nothing

Capacitor



$i = C\,dv/dt$

electric field

$w = \frac{1}{2}Cv^2$

remembers voltage

Inductor



$v = L\,di/dt$

magnetic field

$w = \frac{1}{2}Li^2$

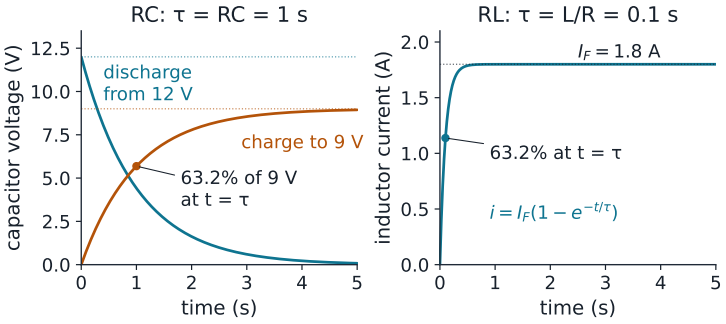
remembers current

The **resistor** burns energy; the **capacitor** and **inductor** store it and remember v

One formula answers every first-order question

1. **Discharge** 100 µF at 12 V through 10 kΩ: $\tau = 1$ s, v = 12 exp(−t); 1.62 V at 2 s.
2. **Charge** Same network to 9 V from 0 V: v = 9(1 − exp(−t)); 5.7 V at 1 s.
3. **Method** Find V_0, V_F, τ - write one line - check both endpoints.

The same shape, capacitor or inductor



Charge and discharge share one curve family; τ marks the 63.2% point.

Interrupting an inductor has consequences

- **Inductor** current is continuous; forcing it to stop makes $v = L\,di/dt$ enormous.
- A 0.1 H coil losing 0.6 A in 1 ms generates 60 V across the opening contact.
- A flyback diode gives the current a path and collapses the spike.

Check yourself

- Quantities: A 3.7 V, 3 A · h battery - energy in J? (About 4×10^4 J.)
- Laws: Node with 2 A and 0.5 A in, 1 A out - the fourth branch? (1.5 A out.)
- Methods: 12 V through 2 Ω to a node with 6 Ω and 1 A in - V_A? (10.5 V.)