

# 19 Circle equations, arcs and unit-circle trigonometry

Name: \_\_\_\_\_ Class: \_\_\_\_\_ Date: \_\_\_\_\_

**Total: 13 marks****KEY WORDS** circle equation • arc length • tangent • radian

## Objective

- Recover circle centre and radius by completing the square
- Convert central angles and calculate arc length and sector area
- Use tangent, inscribed-angle and unit-circle relationships

Original public practice; marks below are classroom self-check points, not official criteria.

## 1 Key knowledge and worked examples

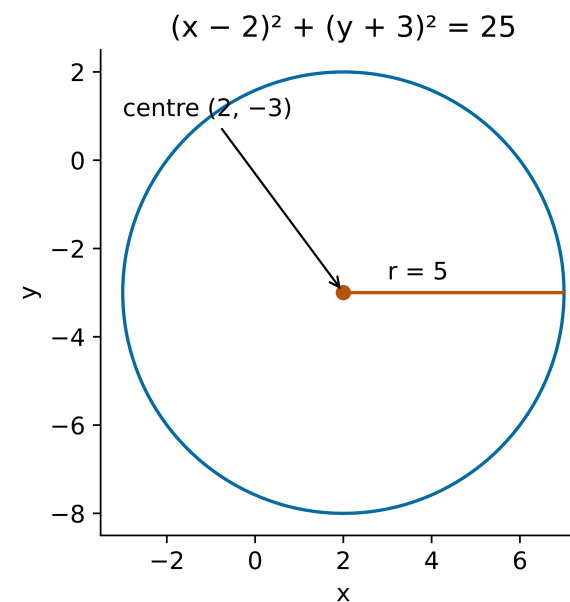
A circle with centre  $(h,k)$  and radius  $r$  satisfies  $(x-h)^2 + (y-k)^2 = r^2$ . For an expanded equation, complete the square in  $x$  and  $y$  and add the balancing constants to the other side. The resulting right side must be positive for a real nondegenerate circle. Keep the radius distinct from its square.

A central angle  $\theta$  in radians gives arc length  $r\theta$  and sector area  $r^2\theta/2$ . In degrees, use the fraction  $\theta/360$  of the full circumference or area. A full turn is  $2\pi$  radians= $360^\circ$ . The formulas  $r\theta$  and  $r^2\theta/2$  require radians; inserting degrees without conversion gives a wrong scale.

A tangent at a circle point is perpendicular to the radius there. An inscribed angle is half the central angle subtending the same arc; an angle subtending a diameter is a right angle. Identify the actual shared arc and centre before applying these statements. A chord is not a tangent and need not be perpendicular to a radius unless further conditions establish it.

On the unit circle, a point at angle  $\theta$  has coordinates  $(\cos \theta, \sin \theta)$ , so  $\cos^2 \theta + \sin^2 \theta = 1$ . Signs depend on quadrant. For acute complementary angles,  $\sin \theta = \cos(90^\circ - \theta)$ . Use special triangles for exact values: at  $30^\circ$  the sine is  $1/2$  and cosine  $\sqrt{3}/2$ ; at  $45^\circ$  both are  $\sqrt{2}/2$ .

Complete both squares in a circle equation.  $x^2 + y^2 + 6x - 8y = 0$  becomes  $(x+3)^2 + (y-4)^2 = 25$ . The centre is  $(-3, 4)$  and radius is 5. Arc length is  $s = r\theta$  only for radian angle  $\theta$ .



*Original worked example from existing native teaching; transfer tasks use their own data.*

Worked example from the native lesson:  $x^2 + y^2 - 4x + 6y - 12 = 0$  becomes  $(x-2)^2 + (y+3)^2 = 25$ , giving centre  $(2, -3)$ , radius 5. For radius 6 and central angle  $\pi/3$ , arc length is  $2\pi$  and sector area  $6\pi$ . An inscribed angle on that same  $60^\circ$  arc is  $30^\circ$ . At  $150^\circ$ , cosine is  $-\sqrt{3}/2$  and sine  $1/2$  because the point is in quadrant II.

## 2 Practice

**2.1** Centre and radius of  $(x+1)^2 + (y-4)^2 = 9$ : [1]

- **A**  $(-1, 4)$ , 3
- **B**  $(1, -4)$ , 9
- **C**  $(-1, 4)$ , 9

**2.2** Radius 4 with central angle  $\pi/2$  gives arc length: [1]

- **A**  $2\pi$
- **B**  $8\pi$
- **C**  $\pi/8$

**2.3** 180 degrees equals \_\_\_\_\_ radians. [1]

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**2.4** The angle between a tangent and its radius at contact is 90 degrees. [1]

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**3 Exam-style questions**

Use the data stated in each original task.

**3.1** Find the centre and radius of  $x^2 + y^2 - 8x + 2y + 8 = 0$ . Does  $(7, -1)$  lie on it? [3]

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**3.2** A sector has radius 8 cm and angle  $135^\circ$ . Find its angle in radians, arc length and area. An inscribed angle elsewhere on the circle intercepts this same arc; find it. [3]

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**3.3** A unit-circle point is in quadrant III and has  $y = -1/2$ . Find its  $x$ -coordinate and one angle between  $0^\circ$  and  $360^\circ$ . [3]

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