Geometry and measures

OxfordAQA International GCSE Mathematics 9260 unit 3.3: geometry and measures, Core and Extension

The recall list

Family	**Recall**
circle	$C = 2\pi r$, $A = \pi r^2$; **sector** $\frac{\theta}{360}$ of each
solids	sphere $\frac{4}{3}\pi r^3$; cone and pyramid $\frac{1}{3}$ of the **prism**
triangles	$a^2 = b^2 + c^2 - 2bc\cos A$; area $\frac{1}{2}ab\sin C$
similarity	lengths k, areas k^2, volumes k^3

Exterior angles walk round once

Regular hexagon: exterior 60°, interior 120°

- Exterior sum 360°.
- Interior $(n - 2) \times 180^\circ$.

One arc, three angles

35° at the **circumference**, 70° at the centre, 145° opposite

- Centre $= 2 \times$ circumference.
- **Cyclic quadrilateral**: opposite angles 180°.

Similar shapes: k, k^2, k^3

> Three times as long: 9 times the area, 27 times the volume

- Lengths k.
- Areas k^2, volumes k^3.

Worked: an angle from two bearings

Angle ABC $= 250° - 160° = 90°$

- Three figures, clockwise from north.
- A north line at every point.

Loci meet where both rules hold

5 from O and equidistant from (0, 0) and (6, 0): $(3, \pm 4)$

- Equidistant: **perpendicular bisector**.
- Fixed distance: a circle.

Worked: a sector

1. **Arc** $\frac{150}{360} \times 2\pi \times 6 = 5\pi = 15.7$ cm
2. **Perimeter** $5\pi + 6 + 6 = 27.7$ cm
3. **Area** $\frac{150}{360} \times \pi \times 6^2 = 15\pi = 47.1$ cm^2

Decision

Both radii count in the perimeter.

A cone from its cross-section

r $= 5$, h $= 12$: slant 13, V $= 100\pi$, curved surface 65π

- Slant by Pythagoras.
- Cone $= \frac{1}{3}$ of the **cylinder**.

Label the triangle first

3, 4, 5: $\tan\theta = 3/4$, $\theta = 36.9°$

- Hypotenuse faces the right angle.
- In 3D, find the right triangle first.

Worked: the cosine rule

1. **Cosine rule** $c^2 = 81 + 121 - 198\cos 72^\circ = 140.8$
2. **Side** $c = 11.9$
3. **Smallest angle** $\cos A = \frac{121+140.8-81}{2\times 11\times 11.87} = 0.693$: $A = 46.2^\circ$

Decision

Check: $46.2^\circ + 61.8^\circ + 72^\circ = 180^\circ$

A vector is across and up

AB $= (4, 3)$, magnitude $\sqrt{(4^2 + 3^2)} = 5$

- Components add and scale.
- Parallel: one is a multiple of the other.

A matrix moves the unit square

Columns (2, 0) and (1, 3): the square becomes a parallelogram

- Columns are the images of (1, 0) and (0, 1).
- Read the **matrix** off the picture.

Worked: a combined transformation

1. **Matrices** reflect in $y = x$: M; rotate 90°: R
2. **Multiply** $RM = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$
3. **Check** $(2, 1) \to (1, 2) \to (-2, 1)$

Decision

Reflection in the y-axis; MR is a different map.

Check yourself

Question	**Answer**
Exterior angle of a regular 10-gon?	$36°$
30° at the **circumference**: centre?	$60°$
Lengths $\times$ 2: volume $\times$?	8
Return bearing of 070°?	$250°$
First of two transformations goes?	on the right