Algebra

OxfordAQA International GCSE Mathematics 9260 unit 3.2: algebra, Core and Extension

The recall list

Family	Recall
quadratic	$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$; $a(x+b)^2 + c$
line	$y - y_1 = m(x - x_1)$, $m = \frac{y_2-y_1}{x_2-x_1}$
calculus	$\frac{d}{dx}(kx^n) = knx^{n-1}$
nth terms	$an + b$; $an^2 + bn + c$

Worked: the subject appears twice

1. **Collect** $ax + 3 = 5xt - 2a$: $ax + 2a = 5xt - 3$
2. **Factorise** $a(x + 2) = 5xt - 3$
3. **Divide** $a = \frac{5xt-3}{x+2}$

Decision

Check every rearrangement with numbers.

Worked: algebraic fractions

1. **Factorise** $\frac{(x-3)(x+3)}{(x+3)(x-2)} = \frac{x-3}{x-2}$
2. **Common denominator** $\frac{3(x-1)-(x+2)}{(x+2)(x-1)}$
3. **Simplify** $\frac{2x-5}{(x+2)(x-1)}$

Decision

Never cancel single terms such as x^2.

Worked: a parity proof

1. **Factorise** $n^2 + n + 9 = n(n+1) + 9$
2. **Consecutive** $n(n+1)$: one factor even
3. **Conclude** even $+ 9$ is odd

Decision

Also: $(2n+1)^2 - (2n-1)^2 = 8n$

Worked: composite and inverse

1. **Composite** $fg(x) = (x-5)^2 + 1 = 5$
2. **Solve** $(x-5)^2 = 4$: $x = 3$ or 7
3. **Inverse** $f(x) = 3x$: $f^{-1}(x) = \frac{x}{3}$

Decision

Check: $(3-5)^2 + 1 = 5$

A line through two points

$m = 2$: $y = 2x + 5$

- $m = \frac{y_2 - y_1}{x_2 - x_1}$.
- **Perpendicular**: $m_1 m_2 = -1$.

Completing the square

$$x^2 - 6x + 5 = (x - 3)^2 - 4$$

- The vertex form shows the **turning point**.
- Roots, intercepts, turning points.

The gradient function

Maximum $(-2, 21)$, minimum $(2, -11)$

- $\frac{dy}{dx}$: gradient of the **tangent**.
- Stationary: $\frac{dy}{dx} = 0$.

Worked: stationary points and a tangent

1. **Derivative** $3x^2 - 12 = 0$: $x = \pm 2$
2. **Classify** $x = -2$: $+ \to -$ max; $x = 2$: $- \to +$ min
3. **Tangent** $m = 15$ at $(3, -4)$: $y + 4 = 15(x - 3)$

Decision

$y = 15x - 49$

9. Equations and inequalities

Equations and inequalities

Quadratics by factorising, completing the square, or the formula: x^2 + 6x + 2 = 0 gives x = -3 +/- sqrt(7) (the 1E specimen family). Pairs: two linear by elimination; linear with quadratic by substitution, keeping both points.

Inequalities: linear in one or two variables and quadratic - x^2 - x - 6 > 0 gives x < -2 or x > 3 (outside the roots). Variation: direct y = kx^n, inverse y = k/x^n (not sampled).

A quadratic inequality

$x^2 - x - 6 > 0$: $x < -2$ or $x > 3$

- Find the critical values.
- Sketch, then read the sign.

A quadratic sequence

nth term $2n^2 - 2n + 5$

- $a = \frac{1}{2} \times$ **second difference**.
- The residuals are linear.

Check yourself

Question	**Answer**
$9x^2 - 25$ factorised?	$(3x-5)(3x+5)$
fg(x): which first?	g
Perpendicular to m $= 2$?	$-\frac{1}{2}$
Stationary point test?	$\frac{dy}{dx} = 0$, then sign
a in $an^2 + bn + c$?	half the **second difference**