Exercise walk-through

Forces ▪ 1.5

eXercise

You must be able to

- describe the load–extension experiment, sketch the graph and use $k = \frac{F}{x}$
- find the resultant of forces on a straight line and use $F = ma$
- describe solid friction and drag, and explain motion in a circle
- use the principle of moments, and state the two conditions for equilibrium
- state what the centre of gravity is, find it for a lamina, and explain stability

Method — Forces change shape, speed or direction

A force can change an object's **shape 形状**, its speed, or its direction. **Solid friction 固体摩擦** is the force between two surfaces that touch: it acts against the motion, tries to stop it, and heats the surfaces. Drag is friction in a liquid or a gas (air resistance). A resultant force at right angles to the motion changes only the direction, so the object moves in a circle at a steady speed. A bigger force is needed for a faster speed or a smaller radius, and for a bigger mass.

Method — Hooke's law and the load–extension experiment

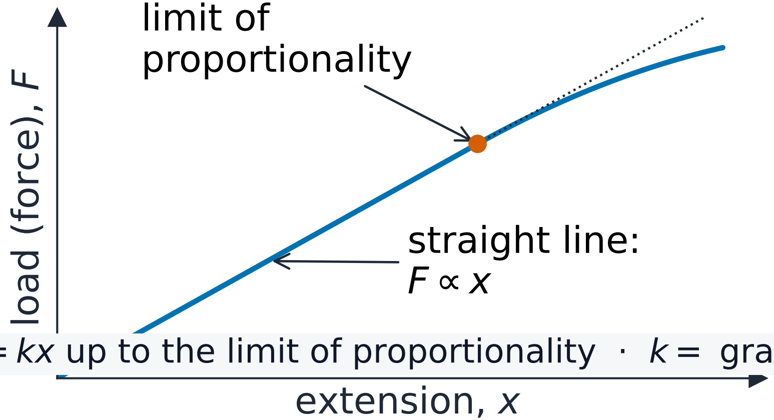
Hang the spring from a clamp with a ruler beside it. Read the position of the bottom of the spring with no load: that is the original length. Add loads one at a time and record the new length each time; the **extension 伸长量** is the new length minus the original length. Plot load (y) against extension (x).

The graph is a straight line through the origin up to the **limit of proportionality 比例极限**, then it curves away. The **spring constant 弹簧常数** is the force per unit extension:

$$k = \frac{F}{x}$$

Worked example 1. A spring stretches 4.0 cm under a load of 20 N. Then $k = \frac{20}{4.0} = 5.0$ N/cm.

Method — Hooke's law and the load–extension experiment



$F = kx$ up to the limit of proportionality · $k =$ gradient

A load–extension graph: straight line then curving past the limit of proportionality

Method — Resultant force and $F = ma$

Add forces on a straight line with signs: one direction positive, the other negative. If the resultant is zero, the object stays at rest or keeps moving in a straight line at constant speed. If it is not zero, the object accelerates in the direction of the resultant:

$$F = ma$$

Worked example 2. A car of mass 1200 kg has a driving force of 3000 N and friction of 600 N.

1. Resultant: $3000 - 600 = 2400$ N.
2. $a = \frac{F}{m} = \frac{2400}{1200} = 2.0 \text{ m/s}^2$.

If the question asks why the engine must produce more force than $F = ma$ suggests, the answer is friction and air resistance: some of the driving force is used against them.

Method — Moments and equilibrium

The **moment 力矩** of a force is its turning effect about a **pivot 支点**:

$$\text{moment} = \text{force} \times \text{perpendicular distance from the pivot}$$

Principle of moments: for a balanced object, total clockwise moment = total anticlockwise moment. An object is in equilibrium when there is **no resultant force and no resultant moment**.

Worked example 3. A 30 N weight sits 0.20 m left of a pivot. How far right must a 20 N weight sit?

$30 \times 0.20 = 20 \times d$, so $d = \dfrac{6.0}{20} = 0.30$ m.

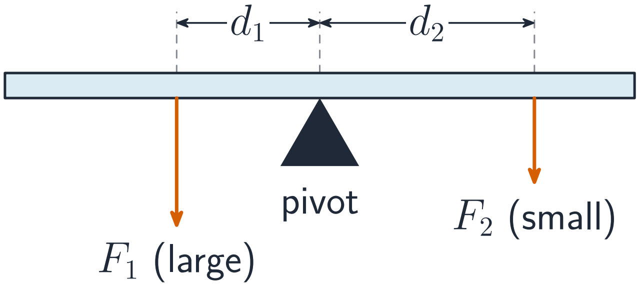
The experiment. Balance a metre rule at its centre. Hang a known weight each side and move them until it balances again. Measure both distances. Force $\times$ distance on

Method — Moments and equilibrium

the left equals force $\times$ distance on the right within the measurement errors, so a balanced object has no resultant moment.

Method — Moments and equilibrium

balanced: $F_1 d_1 = F_2 d_2$



A beam balanced on a central pivot, a force and distance each side

Method — Centre of gravity and stability

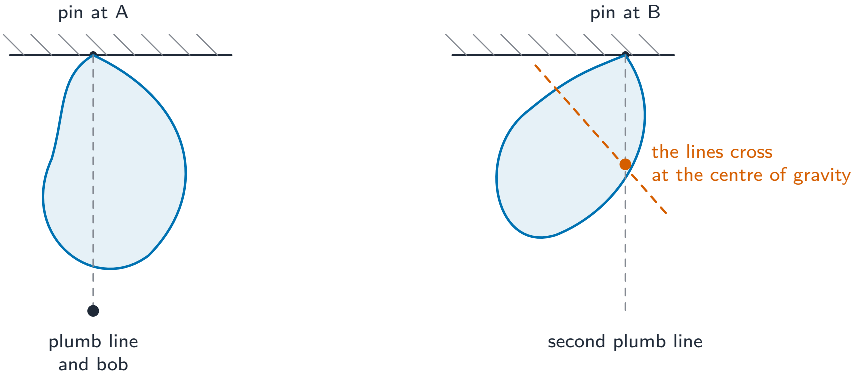
The **centre of gravity** is the single point where all the weight of an object seems to act.

Finding it for a lamina 薄片 (a flat shape): hang the shape from a pin at a point near its edge and let it settle. Hang a plumb line from the same pin and draw the vertical line on the shape. Repeat from a second point. The centre of gravity is where the two lines cross. (A third line is a useful check.)

Stability 稳定性 depends on where the centre of gravity is and how wide the base is. A low centre of gravity and a wide base make an object stable. It tips over once a vertical line down from the centre of gravity falls outside the base.

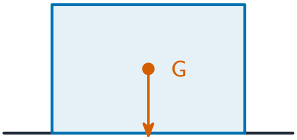
While the weight acts inside the base the object settles back; once it acts outside the pivot corner the object tips over

Method — Centre of gravity and stability

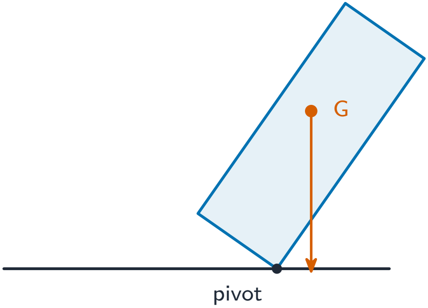


A lamina hung from two points, the plumb lines crossing at the centre of gravity

Method — Centre of gravity and stability



low and wide: the weight acts
well inside the base



tall and narrow: the weight acts
outside the pivot, so it tips over

A wide low block, stable; a tall narrow block tipped past its pivot

Practice 2.1 ▪ 2 marks

A spring stretches 4.0 cm under a 20 N load. Find the spring constant in N/cm.

Solution 2.1

Method: Hooke's law and the load–extension experiment

Worked steps

$$k = \frac{F}{x} = \frac{20}{4.0} = 5.0\ \mathrm{N/cm} \quad \boxed{\textbf{M1}}\boxed{\textbf{A1}}.$$

Practice 2.2 ▪ 1 mark

State the principle of moments.

Solution 2.2

Method: Moments and equilibrium

Worked steps

For a balanced object, the total clockwise moment about the pivot equals the total anticlockwise moment **B1**.

Practice 2.3 ▪ 2 marks

Describe what is meant by solid friction, and state one effect it has besides slowing an object down.

Solution 2.3

Worked steps

The force between two surfaces that touch, which acts against (impedes) the motion **B1**; it also produces heating of the surfaces **B1**.

Practice 2.4 ▪ 2 marks

State the two conditions for an object to be in equilibrium.

Solution 2.4

Worked steps

No resultant force **B1** and no resultant moment **B1**.

Practice 2.5 ▪ 3 marks

A spring of original length 12.0 cm stretches to 15.5 cm under a load of 7.0 N. Calculate its spring constant in N/cm. (Hint: use the extension, not the whole length.)

Solution 2.5

Method: Hooke's law and the load–extension experiment

Worked steps

Extension $= 15.5 - 12.0 = 3.5$ cm **B1**; $k = \dfrac{7.0}{3.5}$ **M1** $= 2.0$ N/cm **A1**.

Practice 2.6 ▪ 1 mark

State what is meant by the centre of gravity of an object.

Solution 2.6

Method: Centre of gravity and stability

Worked steps

The single point at which all of the weight of the object appears to act **B1**.

Practice 2.7 ▪ 2 marks

A ball on the end of a string is whirled in a horizontal circle at a steady speed. State the direction of the resultant force on the ball, and state what the force changes.

Solution 2.7

Method: Forces change shape, speed or direction

Worked steps

Towards the centre of the circle **B1**; it changes the direction of the velocity, not the speed **B1**.

Exam-style 3.1 ▪ 1 mark

A car is travelling along a straight road at a constant speed. Which statement about the forces on the car is correct?

A The driving force is larger than the total resistive force.

B The driving force is equal to the total resistive force.

C There is no driving force and no resistive force.

D The driving force is smaller than the total resistive force.

Solution 3.1

Method: Resultant force and $F = ma$

- **A** The driving force is larger than the total resistive force.
- **B** The driving force is equal to the total resistive force.
- **C** There is no driving force and no resistive force.
- **D** The driving force is smaller than the total resistive force.

Worked steps

B. Constant speed in a straight line means zero resultant force, so the driving force balances the resistive forces. A would make it accelerate and D would make it slow down; C cannot be right because friction and air resistance always act.

Exam-style 3.2 ▪ 1 mark

Two objects, X and Y, have the same mass and the same height. X has a wide base and Y has a narrow base. Which is more stable, and why?

A X, because its centre of gravity is further from the ground.

B X, because the weight stays inside its base for a bigger tilt.

C Y, because it is narrower and lighter at the top.

D They are equally stable, because they have the same mass.

Solution 3.2

Method: Centre of gravity and stability

A X, because its centre of gravity is further from the ground.

B X, because the weight stays inside its base for a bigger tilt. ✓

C Y, because it is narrower and lighter at the top.

D They are equally stable, because they have the same mass.

Worked steps

B. A wide base means the object can tilt further before the vertical line from the centre of gravity leaves the base. A has the reasoning the wrong way round, C is untrue, and D ignores the shape.

Exam-style 3.3 ▪ 1 mark

A train has a total mass of $520\,000$ kg. It accelerates uniformly along a straight, level track.

(a) Define acceleration.

(b) The train speeds up from 15 m/s to 28 m/s with an acceleration of 1.1 m/s^2. Calculate the time this takes.

(c) Calculate the resultant force needed to produce this acceleration.

(d) The force produced by the train's motors is larger than the value in (c). Explain why.

(e) The driver then applies the brakes and the train slows down. State how the direction of the resultant force compares with the direction of motion.

Solution 3.3

Method: Resultant force and \$F = ma\$

Worked steps

(a) The change in velocity per unit time **B1**.

(b) $t = \dfrac{\Delta v}{a} = \dfrac{28 - 15}{1.1}$ **M1** $= 12$ s **A1**.

(c) $F = ma = 520\,000 \times 1.1$ **M1** $= 570\,000$ N **A1** *allow* 5.7×10^5 *N*.

(d) Extra force is needed to overcome friction (or air resistance / drag), because $F = ma$ gives only the resultant force **B1**.

(e) The resultant force is in the opposite direction to the motion **B1**.

Exam-style 3.4 ▪ 1 mark

A uniform beam of weight 12 N and length 1.2 m rests on a pivot at its centre. A 40 N weight hangs 0.30 m to the left of the pivot.
(a) Explain why the weight of the beam itself has no turning effect about this pivot.
(b) A weight W is hung 0.50 m to the right of the pivot so that the beam balances. Calculate W.
(c) The 40 N weight is now moved to 0.45 m from the pivot. State and explain what happens to the beam.
(d) Describe an experiment that shows there is no resultant moment on a balanced beam.

Solution 3.4

Method: Moments and equilibrium

Worked steps

(a) The beam is uniform, so its centre of gravity is at its centre, directly above the pivot; the perpendicular distance is zero, so its moment is zero **B1**.
(b) Clockwise moment = anticlockwise moment: $40 \times 0.30 = W \times 0.50$ **M1**;
$12 = 0.50W$ **M1**; $W = 24$ N **A1**.
(c) The anticlockwise moment becomes $40 \times 0.45 = 18$ N m, which is larger than the clockwise $24 \times 0.50 = 12$ N m **B1**, so the beam turns anticlockwise: the left-hand side goes down **B1**.
(d) Balance a metre rule at its centre on a pivot **B1**; hang a known weight on each

Solution 3.4

side and adjust the distances until it balances, then measure both distances **B1**; show that force $\times$ distance is the same on both sides, within the measurement errors **B1**.

Exam-style 3.5 ▪ 3 marks

A student investigates how a spring stretches.
(a) Describe how she should carry out the experiment, including the measurements she takes.
(b) Sketch the load–extension graph she obtains, continuing the line beyond the limit of proportionality. Label the limit of proportionality on your sketch.
(c) Her spring stretches 2.5 cm for a load of 10 N, within the straight part of the graph. Calculate the extension for a load of 6.0 N.
(d) Explain why her method for the load of 60 N gives an extension larger than the calculation predicts.

Solution 3.5

Method: Hooke's law and the load–extension experiment

Worked steps

(a) Hang the spring from a clamp with a ruler fixed beside it and record the position of the bottom with no load **B1**; add loads one at a time, recording the load and the new length each time **B1**; calculate the extension as new length minus original length **B1**.

(b) A straight line through the origin **B1**; a marked point where the line stops being straight, labelled "limit of proportionality" **B1**; beyond it the line curves so that extension grows faster than load **B1**.

(c) $k = \dfrac{10}{2.5} = 4.0$ N/cm **M1**; $x = \dfrac{F}{k} = \dfrac{6.0}{4.0}$ **M1** $= 1.5$ cm **A1**.

Solution 3.5

(d) A load of 60 N is beyond the limit of proportionality, so extension is no longer proportional to load **B1**.