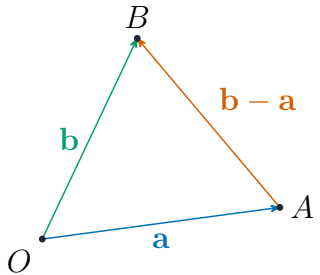
Transformations & Vectors

IGCSE Mathematics ▪ Topic 7

IGCSE Mathematics

What we will cover

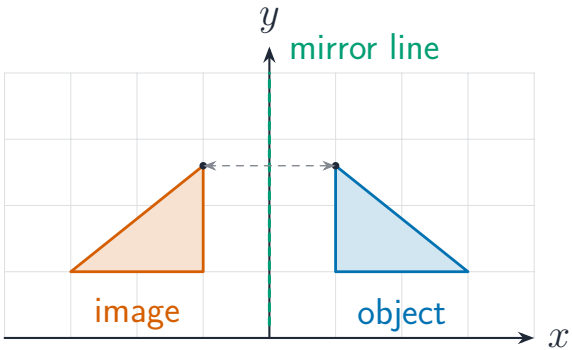
1. Reflection
2. Rotation
3. Enlargement
4. Translation
5. Vectors
6. Magnitude



vector geometry

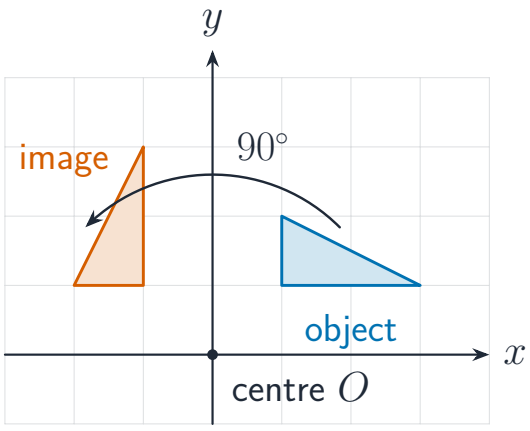
1 Transformations

Reflection



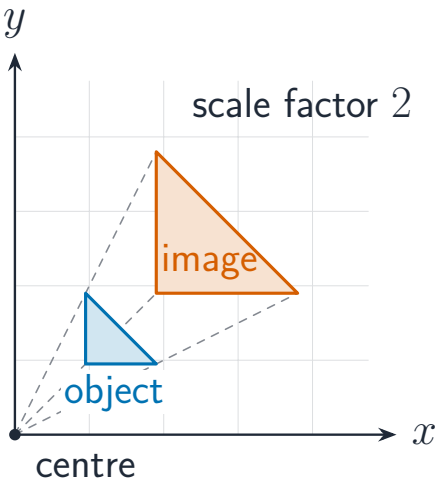
- To describe it, give the **equation of the mirror line** (for Core, a horizontal or vertical line; Extended allows any line such as $y = x$).

Rotation



- To describe it, give the centre, the angle, and the direction (clockwise or anticlockwise).

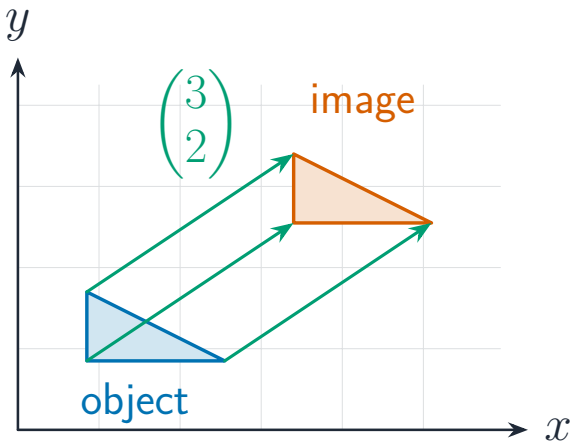
Enlargement



An **enlargement** 放大 scales every distance from a centre by k.

> Give the centre & the scale factor k (k may be fractional or negative).

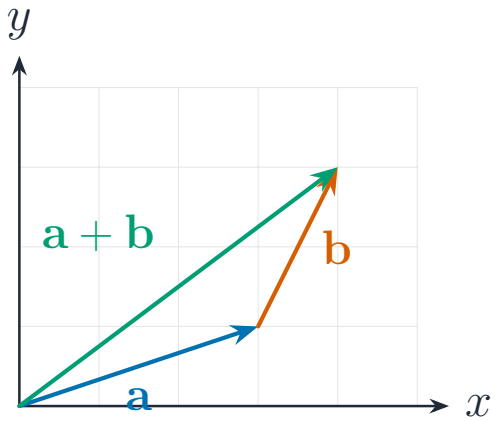
Translation



A **translation** 平移 slides the shape with no turning, by a **column vector** 列向量 $\begin{pmatrix}x\\y\end{pmatrix}$ (x across, y up).

2 Vectors

Vectors



A **vector** 向量 has size & direction.

- add/subtract top & bottom separately
- $\times$ a **scalar** 标量: multiply both

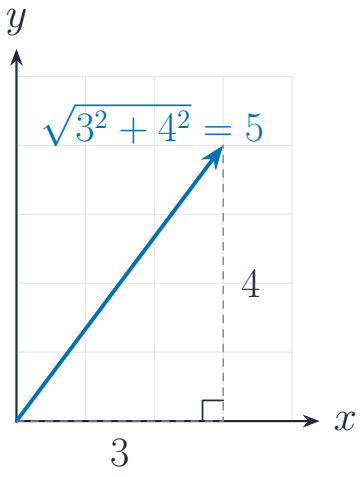
$$\begin{pmatrix}3\\1\end{pmatrix}+\begin{pmatrix}2\\-4\end{pmatrix}=\begin{pmatrix}5\\-3\end{pmatrix}.$$

Two things you can do to a vector

Multiply by a scalar	To **multiply by a scalar** 数乘 stretches the vector and keeps its direction – or reverses it if the scalar is negative.
Add them	Tip-to-tail, or add the components. $\overrightarrow{AB}+\overrightarrow{BC}=\overrightarrow{AC}$, whatever the route.
Parallel	Two vectors are parallel exactly when one is a scalar multiple of the other – which is how a proof of collinearity is written.
Coplanar	Vectors are **coplanar** 共面 when they all lie in one plane, so each can be written from the other two.

To prove three points collinear, show two of the vectors between them are parallel *and* share a point.

Magnitude



The **magnitude** 模 (length) uses Pythagoras:

$$\left|\begin{pmatrix} x \\ y \end{pmatrix}\right| = \sqrt{x^2+y^2}$$

Transformations

Geometric tiles are built by reflecting, rotating and translating shapes.

Vectors in two dimensions (Extended)

Forces like wind are vectors, with both size and direction.

Transformations, and the vector words

a transformation 变换	changes the position or size of a shape: the original is the object, the result the image
what to state	reflection needs a mirror line; rotation needs a centre, angle and direction; enlargement a centre and scale factor; translation a column vector
a directed line segment 有向线段	a vector drawn as an arrow – length is magnitude, direction is direction
a position vector 位置向量	a vector from the origin to a point
multiplying by a scalar	changes the length and, if negative, reverses the direction – but never the line it lies along
collinear 共线 points	if $\overrightarrow{AB}$ is a scalar multiple of $\overrightarrow{BC}$ and they share B, then A, B and C lie on one line

Describing a transformation fully is what is marked. A reflection without its mirror line, or a rotation without its centre, is a half answer.

Exam tips

- Fully **describe** each transformation: a translation (a vector), a reflection (the mirror line), a rotation (centre, angle *and* direction), an enlargement (centre and scale factor).
- A **negative** scale factor turns the image upside down through the centre; a fractional one (between 0 and 1) makes it smaller.
- Add vectors tip-to-tail – add the top numbers, then the bottom numbers. The **magnitude** of a vector comes from Pythagoras on its components.
- $\overrightarrow{AB} = \mathbf{b} - \mathbf{a}$ (end minus start). Two vectors are **parallel** if one is a scalar multiple of the other.

3 Recap

Worked example – reflections

Reflect the point $(3, 2)$ in the y-axis, then in the x-axis.

- In the y-axis (the vertical one): only the sign of x changes.
- $(3, 2) \to (-3, 2)$
- In the x-axis (the horizontal one): only the sign of y changes.
- $(3, 2) \to (3, -2)$

Reflecting in the y-axis flips x; in the x-axis flips y – the opposite letter to the axis named. Points on the mirror line never move.

Topic 7 – key takeaways

1 Reflection

flip over a mirror line

2 Rotation

centre, angle & direction

3 Enlargement

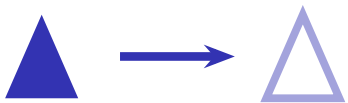
centre & scale factor k

4 Vectors

add componentwise; $|v| = \sqrt{x^2 + y^2}$

Check yourself

1. What must you give to describe a reflection?
2. What three things describe a rotation?
3. What kind of transformation is a slide?
4. Find $\left|\begin{pmatrix}3\\4\end{pmatrix}\right|$.



Answers **1.** the equation of the mirror line **2.** centre, angle, direction **3.** a translation **4.** $\sqrt{9+16}=5$