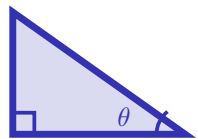
Trigonometry

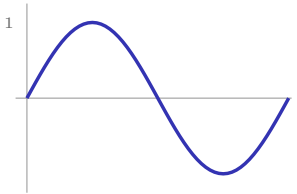
IGCSE Mathematics ▪ Topic 6

IGCSE Mathematics



What we will cover

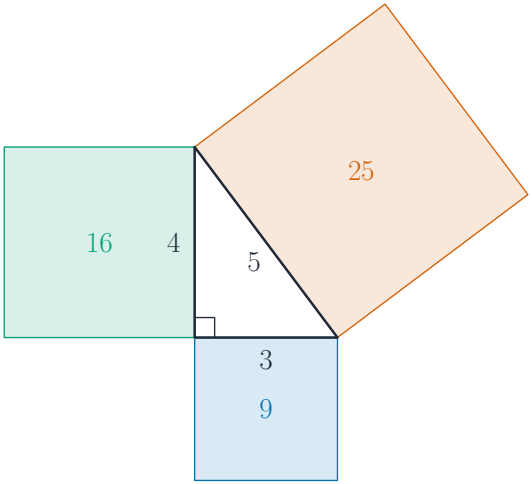
1. Pythagoras' theorem
2. SOH-CAH-TOA
3. Elevation & depression
4. Exact values
5. Trig graphs
6. Sine & cosine rules



trig graphs

1 Right-angled triangles

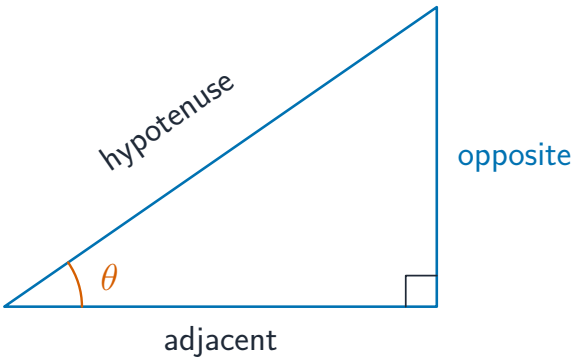
Pythagoras' theorem



$a^2 + b^2 = c^2$ – use it to find a missing side.

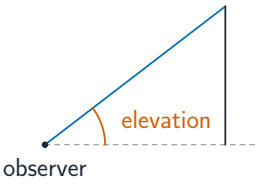
$b^2 = 13^2 - 5^2 = 169 - 25 = 144,$
so $b = \sqrt{144} = 12\,\text{cm}.$

SOH-CAH-TOA

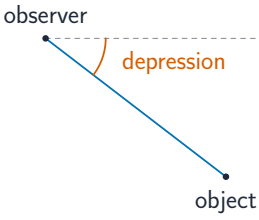


- $y = \sin x$ is a wave starting at 0, peaking at $90°$, back to 0 at $180°$, down to -1 at $270°$.
- $y = \cos x$ is the same wave but starting at 1.
- $y = \tan x$ rises steeply and repeats every $180°$.

Elevation and depression



angle of elevation



angle of depression

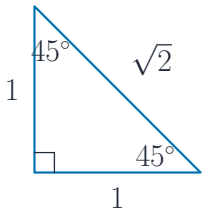
The **angle of elevation** 仰角 looks up; the **angle of depression** 俯角 looks down.

Tower: $50 \tan 40^\circ = 42.0$ m.

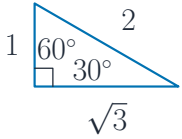
2 Values and graphs

Exact values

	$30°$	$45°$	$60°$
sin	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$
cos	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$
tan	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$



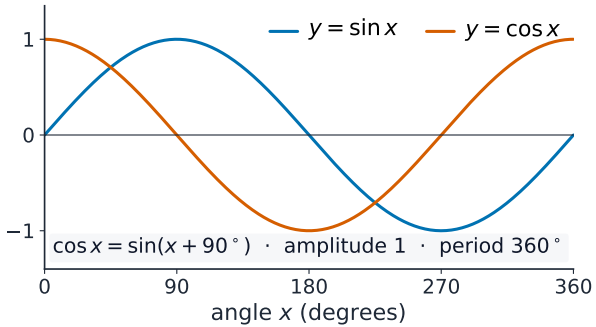
$45°$ **triangle**



$30°$–$60°$ **triangle**

- $y = \sin x$ is a wave starting at 0, peaking at $90°$, back to 0 at $180°$, down to -1 at $270°$.
- $y = \cos x$ is the same wave but starting at 1.
- $y = \tan x$ rises steeply and repeats every $180°$.

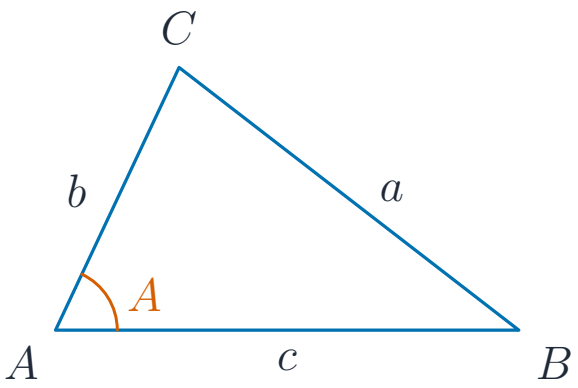
Trig graphs



$y = \sin x$ & $y = \cos x$ are waves between -1 & 1.

A trig equation has several answers in 0–$360°$ – use the wave's symmetry. $\sin x = \frac{\sqrt{3}}{2} \Rightarrow x = 60°$ or $120°$.

Sine and cosine rules



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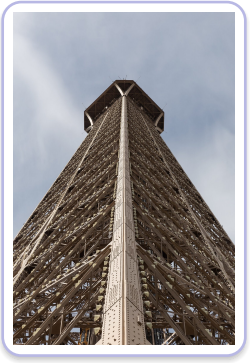
Angles of elevation and depression (Extended)



Looking up at a tower involves an angle of elevation.

Graphs and trigonometric equations (Extended)

A Ferris wheel: a point on the rim traces a sine curve as it turns.

The rules, the ambiguous case, and 3D

the sine rule	$\frac{a}{\sin A} = \frac{b}{\sin B}$ – for a side and its opposite angle
the cosine rule	$a^2 = b^2 + c^2 - 2bc\cos A$ – for two sides and the included angle, or three sides
a worked value	$a^2 = 7^2 + 8^2 - 2(7)(8)\cos 40° = 113 - 112(0.766) = 27.2$, so $a = 5.2$
the ambiguous case 歧义情形	the sine rule can give two valid angles – one acute and one obtuse – when you are given two sides and a non-included angle
the tangent ratio 正切比	$\tan\theta = \frac{\text{opposite}}{\text{adjacent}}$
in 3D	find a right-angled triangle inside the solid, then use Pythagoras or trigonometry on that plane

For a 3D question, draw the triangle on its own before calculating. Working inside the perspective drawing is where the mistakes come from.

Exam tips

- Use **Pythagoras** ($a^2 + b^2 = c^2$) when there is no angle; use **SOH-CAH-TOA** when an angle is involved.
- The **hypotenuse** is always opposite the right angle. Label the sides (opposite, adjacent, hypotenuse) relative to the angle you are using.
- To find an angle, use the inverse ($\sin^{-1}$, $\cos^{-1}$, $\tan^{-1}$), and check your calculator is set to **degrees**.
- For a triangle with no right angle, use the **sine rule** or the **cosine rule** – the cosine rule when you know two sides and the angle between them.

3 Recap

Worked example – Pythagoras

A right-angled triangle has hypotenuse $13\ \text{cm}$ and one short side $5\ \text{cm}$. Find the other short side.

- $a^2 + b^2 = c^2$, where c is the hypotenuse (opposite the right angle).
- $5^2 + b^2 = 13^2$
- $25 + b^2 = 169 \Rightarrow b^2 = 144$
- $b = \sqrt{144} = 12\ \text{cm}$

Finding a shorter side means subtract; finding the hypotenuse means add. Check: the hypotenuse must be the longest side.

Topic 6 – key takeaways

1 Pythagoras

$a^2 + b^2 = c^2$ for a right triangle

2 Ratios

SOH-CAH-TOA; inverse to find an angle

3 Graphs

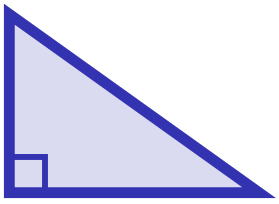
sine & cosine waves; many solutions

4 Any triangle

the sine rule & the cosine rule

Check yourself

1. What is the hypotenuse?
2. Which ratio is opp over adj?
3. What is $\sin 30^\circ$ exactly?
4. Which rule works for a non-right triangle?



Answers **1.** the longest side, opposite the right angle **2.** tangent **3.** $\frac{1}{2}$ **4.** the sine or cosine rule