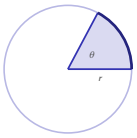
Mensuration

IGCSE Mathematics ▪ Topic 5

IGCSE Mathematics

What we will cover



arcs and sectors

1 Area

Units of measure

- Length: 1 m $= 100$ cm.
- Area: 1 m$^2 = 100^2 = 10\,000$ cm^2.
- Volume: 1 m$^3 = 100^3 = 1\,000\,000$ cm^3.
- Capacity: 1 litre $= 1000$ cm^3, so 1 m$^3 = 1000$ litres.

(to a smaller unit, multiply)

Converting units, squares and cubes included

length	1 m $= 100$ cm $= 1000$ mm
area	square the length factor: 1 m$^2 = 100^2 = 10\,000$ cm^2
volume	cube it: 1 m$^3 = 100^3 = 1\,000\,000$ cm^3
capacity	1 litre $= 1000$ cm^3, and 1 ml $= 1$ cm^3
mass	1 kg $= 1000$ g, 1 tonne $= 1000$ kg
the check	converting to a smaller unit always gives a bigger number

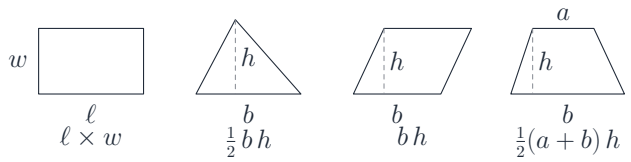
> The commonest error in this whole topic is using the length factor for an area or a volume. Square it or cube it every time.

Area of basic shapes

The **perimeter** 周长 is the distance all the way round a shape.

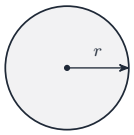
> The **area** is the amount of flat space inside it.

Area of basic shapes, in detail



The **perimeter** 周长 is the distance all the way round a shape.

Circles



circumference $= \pi d$

area $= \pi r^2$

(diameter $d = 2r$)

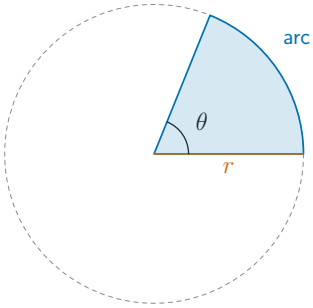
For a **circle** 圆 with **radius** 半径 r (and **diameter** 直径 $d = 2r$):

circumference $= 2\pi r = \pi d$, area $= \pi r^2$.

The **circumference** 圆周 is the distance round...

circumference $= 2\pi \times 7 = 14\pi$ cm, area $= \pi \times 7^2 = 49\pi$ cm^2.

Arcs and sectors



A **sector** 扇形 is the fraction $\frac{\theta}{360}$ of the circle.

$$\text{arc} = \frac{\theta}{360} \times 2\pi r$$

$$\text{sector area} = \frac{\theta}{360} \times \pi r^2$$

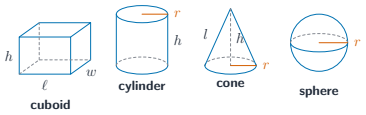
Areas, arcs and sectors

parallelogram 平行四边形	base × perpendicular height
trapezium 梯形	$\frac{1}{2}(a+b)h$, with a and b the two parallel sides
triangle	$\frac{1}{2}bh$, or $\frac{1}{2}ab\sin C$ when you have two sides and the angle between
arc length 弧长	$\frac{\theta}{360} \times 2\pi r$
sector area	$\frac{\theta}{360} \times \pi r^2$
the two sectors	a small slice is a **minor sector** 小扇形; the large rest is a **major sector** 大扇形

Both arc and sector formulas are the whole circle scaled by $\theta/360$. Learn one idea rather than two formulas.

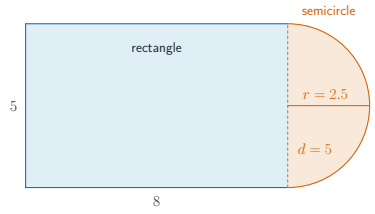
2 Solids

Solids: volume and surface area



- **prism** 棱柱: cross-section × length
- **cylinder** 圆柱: $\pi r^2 h$
- **cone** 圆锥: $\frac{1}{3}\pi r^2 h$
- **sphere** 球: $\frac{4}{3}\pi r^3$

Compound shapes



Split a **compound shape** 复合图形 into parts you know, find each area (or volume), then add them up.

Surface area and volume of solids

The pyramids of Giza are square-based pyramids – a 3-D solid.

The curved surfaces

Cylinder	**Curved surface area** 侧面积 $= 2\pi rh$ – unroll it and it is a rectangle of height h and width the circumference.
Cone	Curved surface $= \pi rl$, where l is the **slant height** 斜高 – not the vertical height. Pythagoras connects the two.
Sphere and hemisphere	Surface $4\pi r^2$, volume $\frac{4}{3}\pi r^3$. A **hemisphere** 半球 is half the volume *plus* a flat circular face.
Frustum	A **frustum** 截头锥体 is a cone with its top cut off: work out the whole cone and subtract the small one.

Check which height a formula wants. Slant height for the cone's surface, vertical height for its volume.

Compound shapes and parts of shapes

Stacked containers are cuboids; volume is length times width times height.

Exam tips

- Match the formula to the shape: the **area** of a circle is πr^2, the **circumference** is πd (or $2\pi r$) – do not mix them up.
- Keep units consistent, and remember area units are squared and volume units cubed (e.g. $1\ \text{m}^2 = 10\,000\ \text{cm}^2$).
- For an **arc** or **sector**, take the fraction $\frac{\theta}{360}$ of the whole circumference or area.
- Surface area is the total of *all* the faces – draw the net if you are unsure. Leave an answer in terms of π only if the question allows it.

3 Recap

Worked example – converting areas

Convert $3\ \text{m}^2$ to cm^2.

- $1\ \text{m} = 100\ \text{cm}$, so square the conversion factor:
- $1\ \text{m}^2 = 100^2 = 10\,000\ \text{cm}^2$
- $3\ \text{m}^2 = 3 \times 10\,000 = 30\,000\ \text{cm}^2$
- For volume, cube it: $1\ \text{m}^3 = 100^3 = 1\,000\,000\ \text{cm}^3$.

The classic error is $\times 100$. Area needs the factor squared, volume needs it cubed – because area is length^2 and volume length^3.

Topic 5 – key takeaways

1 Units
area $\times 100^2$, volume $\times 100^3$ per metre

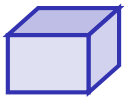
2 Circle
$C = 2\pi r$, area $= \pi r^2$

3 Sector
the fraction $\theta/360$ of the circle

4 Solids
cylinder $\pi r^2 h$, sphere $\frac{4}{3}\pi r^3$

Check yourself

1. How many cm^2 are in $1\ \text{m}^2$?
2. What is the area of a circle of radius r?
3. What fraction of a circle is a 90° sector?
4. What is the volume of a sphere of radius r?



Answers **1.** $10\,000$ **2.** πr^2 **3.** one quarter **4.** $\frac{4}{3}\pi r^3$