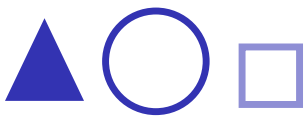
Geometry

IGCSE Mathematics ▪ Topic 4

IGCSE Mathematics

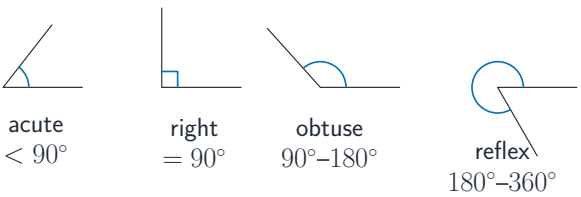


What we will cover

1. Angles & angle facts
2. Parallel-line angles
3. Triangles & polygons
4. Circle parts
5. Circle theorems
6. Bearings & similar shapes

1 Angles

Angle types



- Angles at a point add up to $360°$.
- Angles on a straight line add up to $180°$.
- **Vertically opposite angles** 对顶角 (made by two crossing lines) are equal.

Naming an angle, and naming a triangle

By size — An **acute angle** 锐角 is under 90°, a right angle exactly 90°, an **obtuse angle** 钝角 between 90° and 180°, and a **reflex angle** 优角 more than 180°.

Equilateral — An **equilateral** 等边 triangle: three equal sides, three angles of 60°.

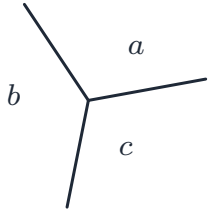
Isosceles — An **isosceles** 等腰 triangle: two equal sides, and the angles opposite them are equal – the fact most angle-chasing questions turn on.

Scalene — A **scalene** 不等边 triangle: all three sides different, so no equal angles to use.

Name the triangle before chasing the angle. Which sides are equal is usually the missing step.

Angle facts

$a + b = 180^\circ$ (straight line)



$a + b + c = 360^\circ$ (round a point)

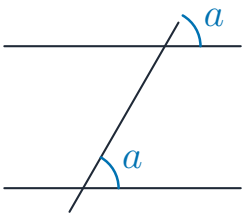
Angles on a line add to 180°; around a point to 360°; **vertically opposite** 对顶角 angles are equal. Always give the reason.

Parallel-line angles

- **Corresponding angles** 同位角 (in matching positions, an "F" shape) are equal.
- **Alternate angles** 内错角 (opposite sides of the crossing line, a "Z" shape) are equal.
- **Co-interior angles** 同旁内角 (a "C" shape) add up to $180°$; we say they are **supplementary** 互补.

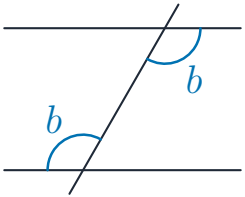
Parallel-line angles, in detail

Corresponding (F)



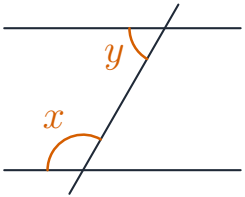
equal

Alternate (Z)



equal

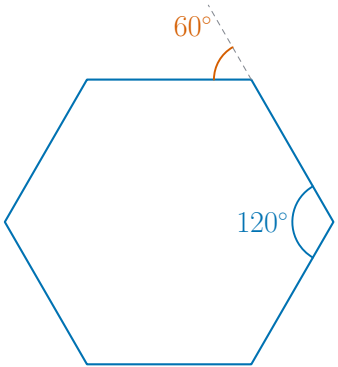
Co-interior (C)



$x + y = 180^\circ$

Corresponding (F) and alternate (Z) angles are equal; co-interior (C) angles add to 180°.

Triangles and polygons

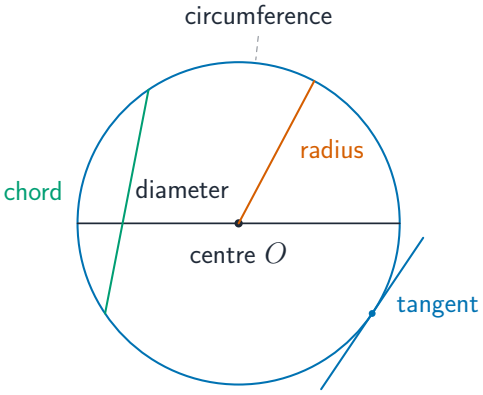


interior + exterior = 180°

- A shape has **line symmetry** 轴对称 if a mirror line splits it into two matching halves.
- A shape has **rotational symmetry** 旋转对称 if it fits onto itself as you turn it. The order is how many times it fits in one full turn.

2 Circles

Circle parts



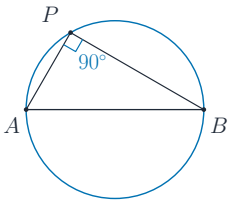
radius 半径, **diameter** 直径, **chord** 弦, **tangent** 切线; a **sector** 扇形 is a slice, a **segment** 弓形 is cut off by a chord.

The parts of a circle, and the theorem names

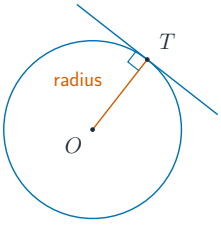
radius 半径	centre to edge (plural *radii*); the diameter is twice it
a chord 弦	a straight line joining two points on the circle
an arc 弧	part of the circumference; a sector is bounded by two radii, a segment by a chord
a tangent 切线	touches at one point, and is perpendicular to the radius there
(Extended) equal chords	are the same distance from the centre
and	the perpendicular bisector of a chord passes through the centre; the **alternate segment theorem 弦切角定理** links a tangent-chord angle to the angle in the other segment

Name the theorem you are using. "Angle in a semicircle is $90°$" scores where "it looks like a right angle" does not.

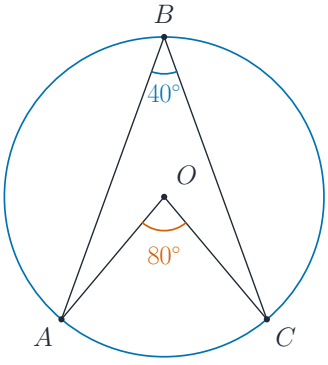
Circle theorems



angle in a semicircle $= 90°$



tangent meets radius $= 90°$

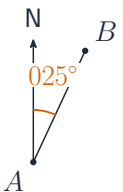


angle at centre $= 2\times$ angle at circumference

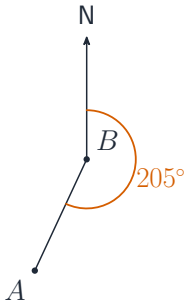
Angle in a **semicircle** 半圆 $= 90°$; tangent$\perp$radius. The angle at the centre is twice the angle at the circumference.

3 Bearings and similarity

Bearings



Bearing of B from A



Bearing of A from B

A **bearing** 方位角 is measured clockwise from north, written with three digits. So due east is $090°$ and south-west is $225°$.

Similar shapes

length	area	volume
$\times k$	$\times k^2$	$\times k^3$

Similar 相似 shapes have the same angles & sides $\times$ a **scale factor** 比例因子 k.
(**Congruent** 全等 = identical.)

Quadrilaterals, symmetry and solids

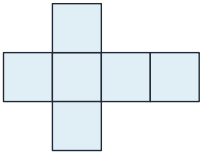
quadrilaterals 四边形
angles sum to $360°$: square, rectangle, rhombus, kite

symmetry 对称
lines of reflection; order of rotational symmetry

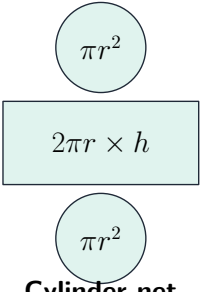
nets & solids 展开图
a net folds into a solid; add the faces for surface area

A **net** 展开图 is the key to surface area – unfold the solid, find each face's area, add them.

Constructions, nets and solids



Cube net
6 equal square faces



Cylinder net
2 circles + a rectangle

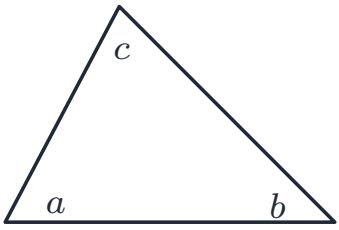
Unfolding a solid gives its net. Add the areas of the pieces to get the surface area: a cylinder is $2\pi r^2 + 2\pi rh$.

Constructions, and the names of solids

to construct 作图 a triangle	draw the base with a ruler, then use **compasses 圆规** to swing the other two side lengths from each end
a scale drawing 比例图	shows a real object smaller or larger by a fixed scale, such as 1 cm to 5 m
prism and cylinder 圆柱	the same cross-section all the way along; a cylinder is a circular prism
pyramid 棱锥 and cone 圆锥	come to a point
sphere 球 and hemisphere 半球	a ball, and half a ball
a frustum 平截头体	a cone or pyramid with the top cut off

A flat side of a solid is a **face** 面, two faces meet at an **edge** 棱, and the whole outside is its **surface** 表面. Leave construction arcs visible – they are marked.

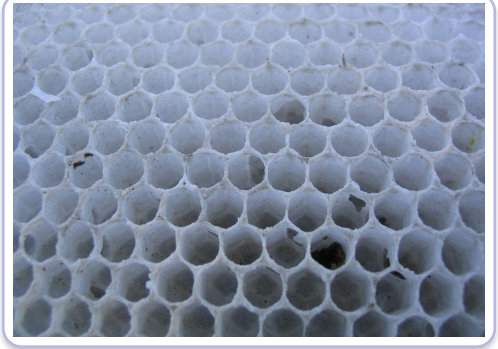
Triangles



$$a + b + c = 180°$$

> The three angles of a triangle add up to 180 degrees

Polygons



A honeycomb tessellates the plane with regular hexagons.

The polygons, and the quadrilaterals

By side count — A **pentagon** 五边形 has 5 sides, a hexagon 6, an **octagon** 八边形 8, a **decagon** 十边形 10. Each corner is a **vertex** 顶点.

Regular polygon — A **regular polygon** 正多边形 has all sides and all angles equal: each exterior angle is $360^\circ/n$, and the interior angle follows.

Parallelogram — A **parallelogram** 平行四边形 has two pairs of parallel sides; opposite sides and opposite angles are equal.

Trapezium — A **trapezium** 梯形 has exactly one pair of parallel sides – which is why its area uses the mean of the two parallel lengths.

Exterior angles of any polygon add to 360°. Start there and the interior angle never has to be memorised.

Similar shapes, continued

Nesting dolls are similar shapes: each one is the same shape as the others, just multiplied by a scale factor

Symmetry

The Taj Mahal has a clear line of symmetry down its centre.

Symmetry, in two dimensions and three

Line symmetry	A shape has a line of symmetry where one half is the mirror image of the other.
Rotational symmetry	The order is how many times it looks the same in one full turn.
Plane of symmetry	In 3D a **plane of symmetry** 对称面 cuts a solid into mirror-image halves: a **cube** 立方体 has nine, a **cuboid** 长方体 three.
Why it helps	A symmetry argument replaces a calculation – it fixes equal lengths and equal angles for free.

Count planes of symmetry by direction: through faces, then through opposite edges. That order stops you missing any.

Exam tips

- Angles on a straight line add to $180°$, around a point to $360°$, and in a triangle to $180°$. Give a **reason** for every step of an angle question.
- Learn the circle theorems: the angle at the centre is *twice* the angle at the circumference; the angle in a semicircle is $90°$; angles in the same segment are equal; opposite angles of a cyclic quadrilateral add to $180°$.
- The **exterior angles** of any polygon add to $360°$, and each interior angle $+$ its exterior angle $= 180°$.
- **Bearings** are measured clockwise from north and always written with three figures (e.g. $072°$).

4 Recap

Worked example – angles on a straight line

Three angles on a straight line are x, 50° and 70°. Find x.

- Angles on a straight line add to 180°.
- $x + 50 + 70 = 180$
- $x + 120 = 180$
- $x = 60^\circ$

Learn the four: straight line 180°, point 360°, triangle 180°, quadrilateral 360°. In an exam, name the rule you used – it carries a mark.

Topic 4 – key takeaways

1 Parallel

F equal, Z equal, C add to $180°$

2 Polygons

interior sum $(n-2) \times 180°$

3 Circles

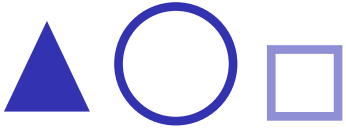
semicircle $90°$; centre $=$ $2\times$ circumference

4 Similar

area $\times k^2$, volume $\times k^3$

Check yourself

1. What do co-interior (C) angles add up to?
2. What is each exterior angle of a regular hexagon?
3. What is the angle in a semicircle?
4. If lengths scale by k, how does volume scale?



Answers **1.** $180°$ **2.** $60°$ **3.** $90°$ **4.** by k^3