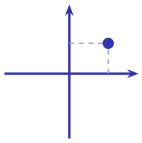
Coordinate Geometry

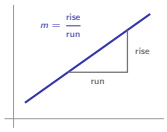
IGCSE Mathematics ▪ Topic 3

IGCSE Mathematics



What we will cover

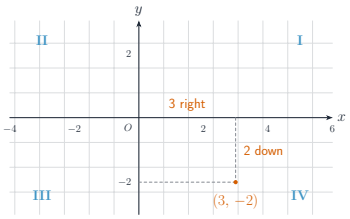
1. Coordinates & axes
2. The line $y = mx + c$
3. Gradient
4. Length & midpoint
5. Parallel & perpendicular



gradient

1 Lines

Coordinates and axes



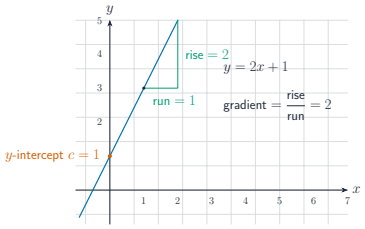
- The two number lines are the **axes** 坐标轴: the **horizontal** 水平 x-axis and the **vertical** 竖直 y-axis.
- They cross at the **origin** 原点, the point $(0, 0)$.
- The axes split the grid into four **quadrants** 象限.

Coordinates, and the perpendicular results

coordinates 坐标	a point is written (x, y) – across first, then up. Sometimes called Cartesian coordinates
a small table of values 数值表	choose two or three x-values, work out y, and plot – enough for a straight line
the gradient	$m = \frac{y_2 - y_1}{x_2 - x_1}$ – rise over run
parallel lines	have the same gradient
perpendicular lines	gradients multiply to -1, so $m_2 = -\frac{1}{m_1}$ – they meet at a **right angle** 直角
the perpendicular bisector 垂直平分线	passes through the midpoint of a segment at right angles to it

The perpendicular bisector needs both results at once: the midpoint for a point on the line, and the negative reciprocal for its gradient.

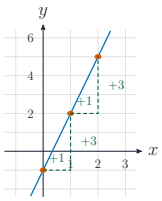
The line $y = mx + c$



$y = mx + c$: m is the **gradient** 斜率, c is the **intercept** 截距 (where it crosses the y-axis).

Rearrange $ax + by = c$ into $y = mx + c$ to read off m & c.

Drawing a line fast



drawing $y = 3x - 1$:

1. mark the intercept $c = -1$
2. gradient 3: step 1 right, 3 up, again and again
3. join the points

check with a table: $x = 0, 1, 2 \;\to\; y = -1, 2, 5$

Drawing a line fast: mark the intercept, then use the gradient to step to new points

Gradient

$$m = \frac{\text{change in } y}{\text{change in } x} = \frac{\text{rise}}{\text{run}}$$

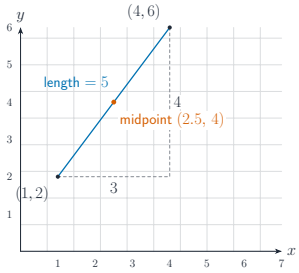
Through $(1, 2)$ and $(4, 11)$

$$m = \frac{11 - 2}{4 - 1} = \frac{9}{3} = \mathbf{3}$$

A steep mountain road: gradient measures steepness as rise over run.

2 Segments

Length and midpoint



A **line segment** 线段 is the straight piece between two points.

To find the **length** 长度 of the segment between (x_1, y_1) and (x_2, y_2), use **Pythagoras' theorem** 勾股定理 on the horizontal and vertical gaps:

$$\text{length} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}.$$

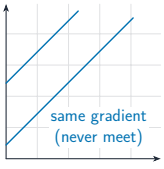
To find the...

Parallel and perpendicular

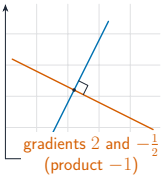
- **parallel** 平行 – same gradient
- **perpendicular** 垂直 – $m_1 \times m_2 = -1$

So flip the fraction & change the sign.

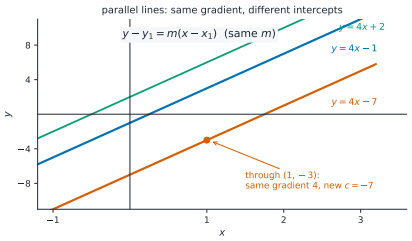
Parallel



Perpendicular



Parallel lines



Parallel lines: same gradient, different intercepts

Coordinates

A city street grid: every place is fixed by its coordinates.

Exam tips

- The straight line is $y = mx + c$: m is the **gradient** and c is where the line crosses the y-axis.
- Gradient $=$ (change in y) $\div$ (change in x). Keep the two coordinates in the same order on the top and the bottom.
- **Parallel** lines have the *same* gradient; **perpendicular** lines have gradients that multiply to -1 (the negative reciprocal).
- The midpoint is the average of the coordinates; the distance between two points comes from Pythagoras on the differences.

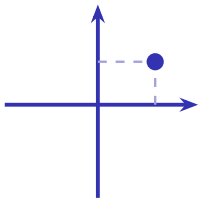
3 Recap

Topic 3 – key takeaways

1. **Line**
   $y = mx + c$: gradient m, intercept c
2. **Gradient**
   rise over run
3. **Length**
   Pythagoras on the gaps; midpoint averages
4. **Lines**
   parallel: same m; perpendicular: $m_1 m_2 = -1$

Check yourself

1. In $y = 4x - 1$, what is the gradient?
2. Find the gradient through $(1, 2)$ and $(4, 11)$.
3. What is the midpoint of $(1, 2)$ and $(4, 6)$?
4. What do perpendicular gradients multiply to?



Answers **1.** 4 **2.** 3 **3.** $(2.5, 4)$ **4.** -1