

E6.4 Trigonometric functions

Name: _____ Class: _____ Date: _____

Total: 11 marks

KEY WORDS sine and cosine rules 正弦和余弦定理 • exact values 精确值 • trigonometric equation 三角方程
• trigonometric functions 三角函数 • sine 正弦

Objective

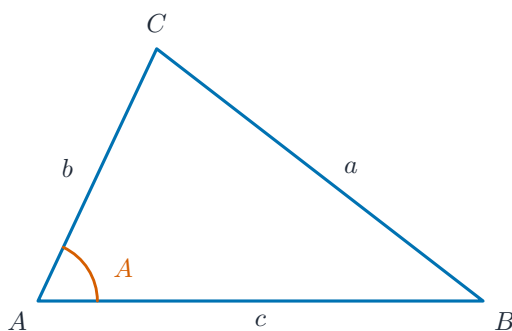
Build the skills to answer Extended exam questions on **trigonometric functions** 三角函数 — **exact values** 精确值, solving **trigonometric equations** 三角方程, and the **sine and cosine rules** 正弦和余弦定理.

You must be able to:

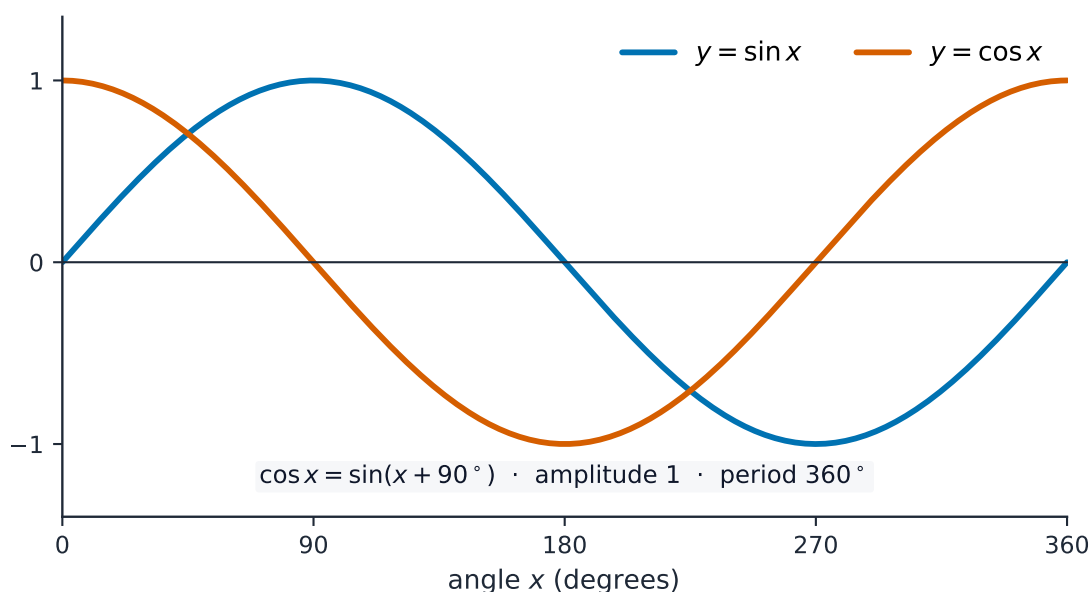
- recall exact values of $\sin$, $\cos$, $\tan$ for $0-90^\circ$
- use the cosine rule and the sine rule in any triangle
- find the area of a triangle with $\frac{1}{2}ab \sin C$

1 Worked examples

■ The sine and cosine rules



For any triangle, not just right-angled ones



The sine and cosine function graphs

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} \quad a^2 = b^2 + c^2 - 2bc \cos A \quad \text{area} = \frac{1}{2}ab \sin C$$

- **Cosine rule (two sides + included angle):** $b = 7$, $c = 8$, $A = 40^\circ \rightarrow a^2 = 49 + 64 - 112 \cos 40^\circ = 27.2 \rightarrow a = 5.2$ cm.
- **Exact:** $\sin 60^\circ = \frac{\sqrt{3}}{2}$; solve $\sin x = \frac{\sqrt{3}}{2}$ ($0-360^\circ$) $\rightarrow x = 60^\circ$ or 120° .

Answer shapes: **M1** a correct method, **A1** the correct answer.

2 Practice

2.1 Write down the exact value of $\cos 30^\circ$. [1]

2.2 Solve $\sin x = \frac{1}{2}$ for $0^\circ \leq x \leq 360^\circ$. [2]

2.3 Find the area of a triangle with sides $a = 6$ cm, $b = 9$ cm and included angle $C = 50^\circ$. [2]

3 Exam-style questions

3.1 A triangle has $b = 5$ cm, $c = 7$ cm and the angle between them $A = 60^\circ$. Find side a . [3]

3.2 In a triangle, $a = 8$ cm, angle $A = 40^\circ$ and angle $B = 75^\circ$. Use the sine rule to find side b . [3]

Solutions

Marking: **M1** a correct method, **A1** the correct answer.

2.1 $\cos 30^\circ = \frac{\sqrt{3}}{2}.$

2.2 $x = 30^\circ$ or $x = 180 - 30 = 150^\circ.$

2.3 $\text{area} = \frac{1}{2}(6)(9) \sin 50^\circ = 27 \times 0.766 = 20.7 \text{ cm}^2.$

3.1 $a^2 = 5^2 + 7^2 - 2(5)(7) \cos 60^\circ = 25 + 49 - 70(0.5) = 39; a = \sqrt{39} = 6.24 \text{ cm}.$

3.2 $\frac{b}{\sin 75^\circ} = \frac{8}{\sin 40^\circ} \rightarrow b = \frac{8 \sin 75^\circ}{\sin 40^\circ} = \frac{8 \times 0.966}{0.643} = 12.0 \text{ cm}.$