Exercise walk-through

Transformations ▪ E7.1

eXercise

You must be able to

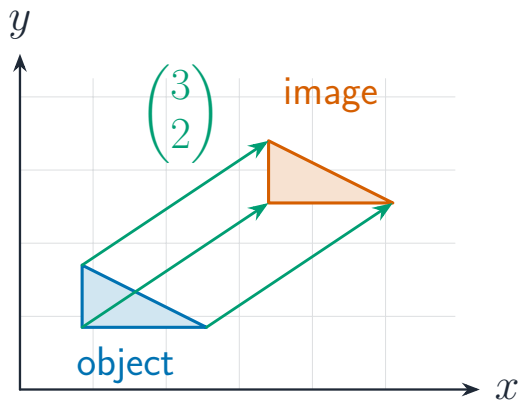
- find the image of a point under each transformation
- describe a transformation with all the required details
- use a column vector for a translation

Method — The four transformations

- **Reflection 反射**: give the mirror line. $(3, 2)$ in the y-axis $\to (-3, 2)$.
- **Rotation 旋转**: give centre, angle, direction. $(3, 1)$, $90°$ anticlockwise about $O \to (-1, 3)$.
- **Enlargement 放大**: give centre and scale factor k. $(1, 2)$ from O, $k = 2 \to (2, 4)$.
- **Translation 平移**: give the column vector. $(5, 3)$ by $\begin{pmatrix} -2 \\ 4 \end{pmatrix} \to (3, 7)$.

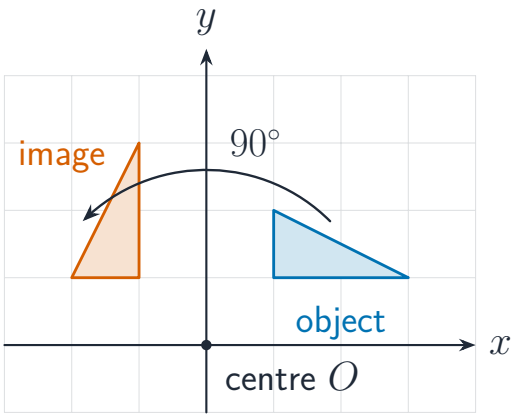
Answer shapes: **M1** a correct method, **A1** the correct answer.

Method — The four transformations



A translation slides every point by the same column vector

Method — The four transformations



Rotation is one of the four transformations

Method — Describing a transformation fully

"Describe fully" means give the **name** and every piece of information that fixes it. Miss one and the marks go.

Transformation	**What must also be given**
reflection	the equation of the mirror line, for example $y = x$ or $x = 2$
rotation	the angle, the direction (clockwise or anticlockwise) and the centre
translation	the column vector $\begin{pmatrix} a \\ b \end{pmatrix}$
enlargement	the scale factor and the centre of enlargement

Method — Describing a transformation fully

Only **one** transformation may be named: writing "a reflection and then a translation" scores nothing.
How to identify it. If the shape is the same size and the same way round, it is a translation or a rotation; if it is the same size but reversed, it is a reflection; if it is a different size, it is an enlargement. For a rotation, join a point to its image and to a candidate centre and check the angle. For an enlargement, the centre is where the lines through matching pairs of vertices meet, and the scale factor is $\dfrac{\text{image length}}{\text{object length}}$.
Worked example. A has vertices $(1, 2)$, $(4, 2)$, $(1, 4)$ and B has $(2, 4)$, $(8, 4)$, $(2, 8)$. Describe the transformation fully.

1. The image is bigger, so it is an enlargement.
2. Scale factor: the base goes from 3 to 6, so it is 2.
3. Centre: each image coordinate is exactly twice the object coordinate, so the centre is the origin $(0, 0)$.
4. Answer: an enlargement, scale factor 2, centre $(0, 0)$.

Method — Describing a transformation fully

A **negative scale factor** puts the image on the other side of the centre and turns it upside down; a **fractional scale factor** makes it smaller.

Practice 2.1 ▪ 1 mark

Reflect the point $(4, 3)$ in the x-axis.

Solution 2.1

Worked steps

$(4, -3)$ B1.

Practice 2.2 ▪ 1 mark

Translate the point $(2, 1)$ by the vector $\begin{pmatrix} 3 \\ -2 \end{pmatrix}$.

Solution 2.2

Method: Describing a transformation fully

Worked steps

$(2+3,\ 1-2) = (5, -1)$ B1.

Practice 2.3 ▪ 1 mark

Enlarge the point $(2, 3)$ from the origin by scale factor 2.

Solution 2.3

Method: Describing a transformation fully

Worked steps

$(2 \times 2,\ 3 \times 2) = (4, 6)$ B1.

Practice 2.4 ▪ 2 marks

Describe fully the transformation that maps $(3, 2)$ onto $(-3, 2)$.

Solution 2.4

Method: Describing a transformation fully

Worked steps

A reflection **B1** in the y-axis, the line $x = 0$ **B1**.

Practice 2.5 ▪ 2 marks

State the two things you must give to describe a rotation fully, besides its name.

Solution 2.5

Method: Describing a transformation fully

Worked steps

The angle of turn and the direction B1, and the centre of rotation B1.

Exam-style 3.1 ▪ 2 marks

The point $(4, 1)$ is rotated 90° anticlockwise about the origin. Find the image.

Solution 3.1

Method: Describing a transformation fully

Worked steps

the rule is $(x, y) \to (-y, x)$, so $(4, 1) \to (-1, 4)$ M1 A1 *for the rule or a correct turn; for* $(-1, 4)$.

Exam-style 3.2 ▪ 3 marks

A quadrilateral has vertices $(2, 1)$, $(5, 1)$, $(5, 3)$ and $(2, 3)$. After one transformation its image has vertices $(6, 3)$, $(15, 3)$, $(15, 9)$ and $(6, 9)$. State the name of this transformation and give the two further pieces of information needed to describe it fully.

Solution 3.2

Method: Describing a transformation fully

Worked steps

An enlargement **B1**, scale factor 3 **B1**, centre $(0, 0)$ **B1**. (Each image coordinate is three times the object coordinate, so the centre is the origin.)

Exam-style 3.3 ▪ 2 marks

A shape is enlarged from the origin by scale factor $\frac{1}{2}$. The point $(6, 4)$ is on the shape. Find its image.

Solution 3.3

Method: Describing a transformation fully

Worked steps

$\left(6 \times \frac{1}{2},\ 4 \times \frac{1}{2}\right) = (3, 2)$ M1 A1 *for multiplying by* $\frac{1}{2}$*; for* $(3, 2)$.

Exam-style 3.4 ▪ 3 marks

Triangle P has vertices $(2, 1)$, $(6, 1)$ and $(2, 3)$.
(a) Triangle P is reflected in the line $x = 1$ to give triangle Q. Write the coordinates of the vertices of Q.
(b) Triangle P is enlarged with centre $(0, 0)$ and scale factor $\frac{3}{2}$ to give triangle R. Write the coordinates of the vertices of R.
(c) Triangle P is rotated $90°$ clockwise about the origin to give triangle S. Write the coordinates of the vertices of S.
(d) Triangle T has vertices $(-2, -1)$, $(-6, -1)$ and $(-2, -3)$. Describe fully the single transformation that maps P onto T.

Solution 3.4

Method: Describing a transformation fully

Worked steps

(a) Reflecting in $x = 1$ sends (x, y) to $(2 - x, y)$ M1; $(0, 1)$, $(-4, 1)$ and $(0, 3)$ A1 A1.
(b) Multiply each coordinate by $\frac{3}{2}$ M1; $(3, 1.5)$, $(9, 1.5)$ and $(3, 4.5)$ A1 A1.
(c) A $90°$ clockwise rotation about the origin sends (x, y) to $(y, -x)$ M1; $(1, -2)$, $(1, -6)$ and $(3, -2)$ A1 A1.
(d) A rotation B1 of $180°$ B1 about the origin $(0, 0)$ B1. (An enlargement of scale factor -1, centre the origin, is also accepted.)