# Exercise walk-through

## Trigonometric functions ▪ E6.4

eXercise

# You must be able to

- recall exact values of sin, cos, tan for $0$–$90^\circ$
- use the cosine rule and the sine rule in any triangle
- find the area of a triangle with $\frac{1}{2}ab\sin C$

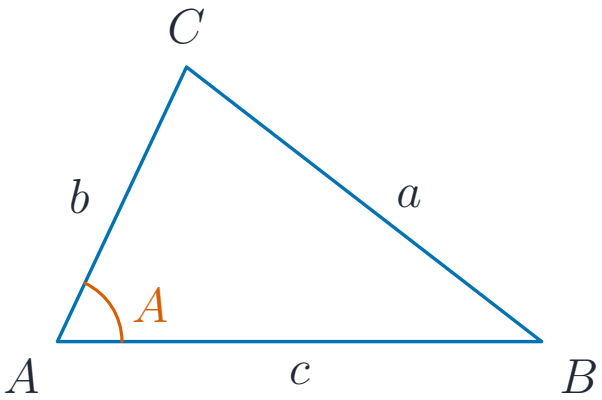
## Method — The sine and cosine rules

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} \qquad a^2 = b^2 + c^2 - 2bc\cos A \qquad \text{area} = \tfrac{1}{2}ab\sin C$$

- **Cosine rule (two sides + included angle):** $b = 7$, $c = 8$, $A = 40^\circ$ $\rightarrow$ $a^2 = 49 + 64 - 112\cos 40^\circ = 27.2$ $\rightarrow$ $a = 5.2$ cm.
- **Exact:** $\sin 60^\circ = \frac{\sqrt{3}}{2}$; solve $\sin x = \frac{\sqrt{3}}{2}$ $(0\text{–}360^\circ)$ $\rightarrow$ $x = 60^\circ$ or $120^\circ$.

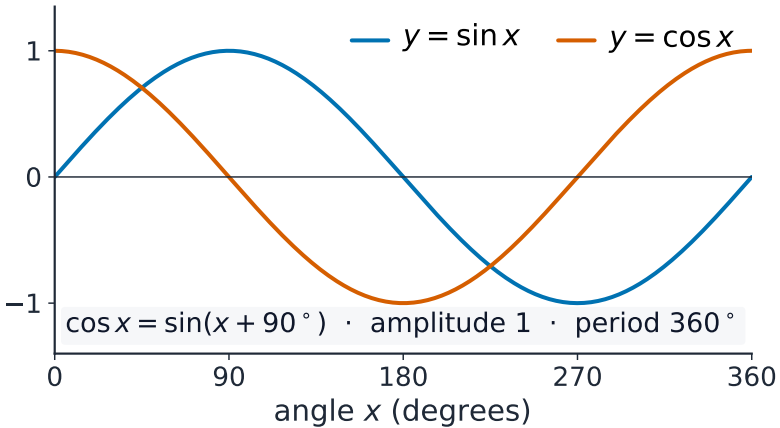
**Answer shapes:** **M1** a correct method, **A1** the correct answer.

# Method — The sine and cosine rules




*In any triangle, sides a, b, c lie opposite angles A, B, C*

# Method — The sine and cosine rules




*The sine and cosine function graphs*

## Practice 2.1 ▪ 1 mark

Write down the exact value of $\cos 30°$.

# Solution 2.1

Method: The sine and cosine rules

**Worked steps**

$\cos 30^\circ = \dfrac{\sqrt{3}}{2}$ B1.

## Practice 2.2 ▪ 2 marks

Solve $\sin x = \frac{1}{2}$ for $0^\circ \leqslant x \leqslant 360^\circ$.

# Solution 2.2

Method: The sine and cosine rules

**Worked steps**

$x = 30^\circ$ or $x = 180 - 30 = 150^\circ$ M1 A1 *for* $30^\circ$*; for both.*

## Practice 2.3 ▪ 2 marks

Find the area of a triangle with sides $a = 6$ cm, $b = 9$ cm and included angle $C = 50°$.

# Solution 2.3

Method: The sine and cosine rules

**Worked steps**

area $= \frac{1}{2}(6)(9)\sin 50^\circ = 27 \times 0.766 = 20.7$ cm$^2$ **M1** **A1** *for* $\frac{1}{2}ab\sin C$*; for* $20.7$.

## Exam-style 3.1 ▪ 3 marks

A triangle has $b = 5$ cm, $c = 7$ cm and the angle between them $A = 60^\circ$. Find side $a$.

# Solution 3.1

Method: The sine and cosine rules

**Worked steps**

$a^2 = 5^2 + 7^2 - 2(5)(7)\cos 60° = 25 + 49 - 70(0.5) = 39$; $a = \sqrt{39} = 6.24$ cm

M1 M1 A1 *for correct substitution; for* $a^2 = 39$*; for* $6.24$.

## Exam-style 3.2 ▪ 3 marks

In a triangle, $a = 8$ cm, angle $A = 40^\circ$ and angle $B = 75^\circ$. Use the sine rule to find side $b$.

# Solution 3.2

Method: The sine and cosine rules

**Worked steps**

$$\frac{b}{\sin 75^\circ} = \frac{8}{\sin 40^\circ} \rightarrow b = \frac{8 \sin 75^\circ}{\sin 40^\circ} = \frac{8 \times 0.966}{0.643} = 12.0 \text{ cm}$$ M1 M1 A1 *for the sine-rule set-up; for rearranging; for* $12.0$.