

Exercise walk-through

Equations ■ E2.5

eXercise

You must be able to

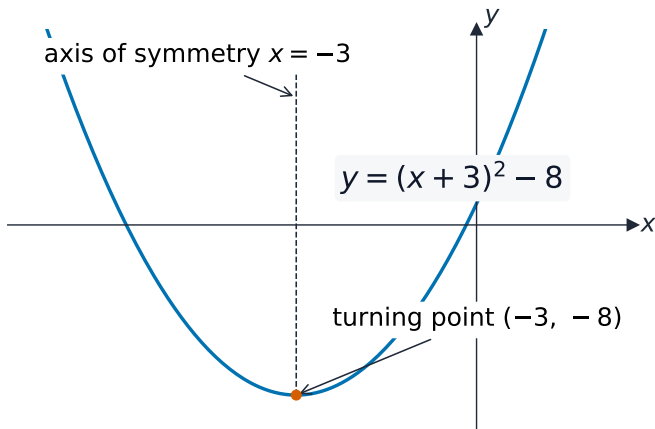
- solve a linear equation with brackets
- solve simultaneous linear equations
- solve a quadratic by factorising or the formula

Method — Solving equations

- **Linear:** $5 - 2x = 3(x + 7) \rightarrow -16 = 5x \rightarrow x = -\frac{16}{5}$.
- **Simultaneous:** $2x + y = 7$, $3x - y = 8 \rightarrow$ add: $5x = 15$, $x = 3$, $y = 1$.
- **Quadratic formula:** $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$.

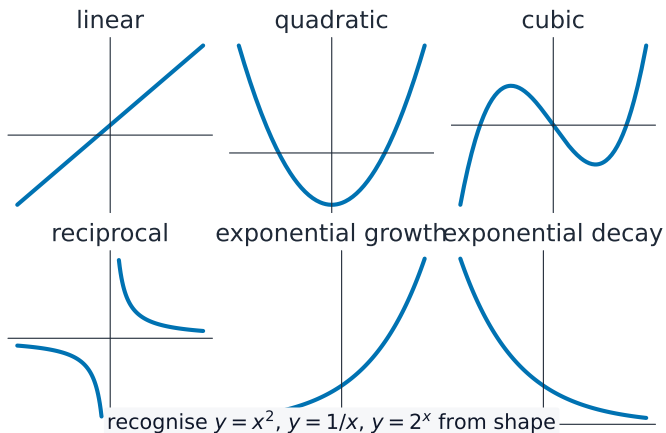
Answer shapes: **M1** a correct method, **A1** the correct answer.

Method — Solving equations



Completing the square gives the turning point

Method — Solving equations



An equation is solved graphically where curves cross

Method — The three routes through a quadratic, and how to pick one

Try them in this order.

- 1 Factorising.** Find two numbers that multiply to give the constant and add to give the coefficient of x . For $x^2 - 2x - 8 = 0$ they are -4 and 2 , so $(x - 4)(x + 2) = 0$ and $x = 4$ or $x = -2$. Fastest when the numbers are whole.
- 2 The formula.** Use it whenever the question says “correct to 2 decimal places”, because that is a sign the roots are not whole numbers:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

- 3 Completing the square.** Use it when the question asks for the turning point or for an exact (surd) answer.

Method — The three routes through a quadratic, and how to pick one

Worked example (formula). Solve $2x^2 + 3x - 4 = 0$ to 2 decimal places. Here $a = 2$, $b = 3$, $c = -4$.

$$1 \quad b^2 - 4ac = 9 - 4(2)(-4) = 9 + 32 = 41.$$

$$2 \quad x = \frac{-3 \pm \sqrt{41}}{4}.$$

$$3 \quad \sqrt{41} = 6.403, \text{ so } x = \frac{3.403}{4} = 0.85 \text{ or } x = \frac{-9.403}{4} = -2.35.$$

Show the substituted formula before evaluating: that line earns the method mark even if the arithmetic slips.

Simultaneous equations. With one linear and one quadratic, **substitute** the linear into the quadratic; with two linear equations, **eliminate** by adding or subtracting once the coefficients match. Always find both unknowns and pair them correctly.

Practice 2.1 ■ 2 marks

Solve $3x - 7 = 11$.

Solution 2.1

Worked steps

$3x = 18 \rightarrow x = 6$ **M1** **A1** for $3x = 18$; for 6.

Practice 2.2 ■ 2 marks

Solve $x^2 + 5x + 6 = 0$.

Solution 2.2

Worked steps

$$(x+2)(x+3) = 0 \rightarrow x = -2 \text{ or } x = -3 \quad \text{M1} \quad \text{A1} \quad \text{for the factors; for both.}$$

Practice 2.3 ■ 2 marks

Solve $4(x - 1) = 2x + 6$.

Solution 2.3

Worked steps

$$4x - 4 = 2x + 6 \rightarrow 2x = 10 \rightarrow x = 5 \quad \text{M1} \quad \text{A1} \quad \text{for } 2x = 10; \text{ for } 5.$$

Practice 2.4 ■ 2 marks

Solve $x^2 - 7x + 12 = 0$ by factorising.

Solution 2.4

Method: The three routes through a quadratic, and how to pick one

Worked steps

$$(x - 3)(x - 4) = 0 \quad \text{M1}; x = 3 \text{ or } x = 4 \quad \text{A1}.$$

Practice 2.5 ■ 1 mark

Write down the values of a , b and c for $3x^2 - 5x - 2 = 0$.

Solution 2.5

Worked steps

$$a = 3, b = -5, c = -2 \quad \text{B1}.$$

Exam-style 3.1 ■ 3 marks

Solve the simultaneous equations $3x + 2y = 12$ and $x - 2y = 4$.

Solution 3.1

Method: The three routes through a quadratic, and how to pick one

Worked steps

add the two equations: $4x = 16 \rightarrow x = 4$; then $4 - 2y = 4 \rightarrow y = 0$ **M1** **A1** **A1**
for eliminating y ; for $x = 4$; for $y = 0$.

Exam-style 3.2 ■ 2 marks

Solve $x^2 - 2x - 8 = 0$.

Solution 3.2

Worked steps

$$(x - 4)(x + 2) = 0 \rightarrow x = 4 \text{ or } x = -2 \quad \text{M1} \quad \text{A1} \quad \text{for the factors; for both.}$$

Exam-style 3.3 ■ 3 marks

Solve $2x^2 + 3x - 4 = 0$, giving your answers correct to 2 decimal places.

Solution 3.3

Method: The three routes through a quadratic, and how to pick one

Worked steps

$a = 2, b = 3, c = -4$: $x = \frac{-3 \pm \sqrt{9+32}}{4} = \frac{-3 \pm \sqrt{41}}{4} \rightarrow x = 0.85$ or $x = -2.35$ **M1** **M1** **A1**
for correct substitution; for $\sqrt{41}$; for 0.85 and -2.35 .

Exam-style 3.4 ■ 5 marks

- (a) Solve the simultaneous equations $y = x + 3$ and $x^2 + y^2 = 29$.
- (b) Solve $3x^2 - 7x + 2 = 0$ by factorising.
- (c) Solve $x^2 + 6x + 4 = 0$, giving your answers correct to 2 decimal places.

Solution 3.4

Method: The three routes through a quadratic, and how to pick one

Worked steps

(a) Substitute $y = x + 3$: $x^2 + (x + 3)^2 = 29$ **M1**; $2x^2 + 6x + 9 = 29$, so $2x^2 + 6x - 20 = 0$ and $x^2 + 3x - 10 = 0$ **M1**; $(x + 5)(x - 2) = 0$ **M1**; $x = -5$ or $x = 2$ **A1**; the matching values are $y = -2$ and $y = 5$ **A1**.

(b) $(3x - 1)(x - 2) = 0$ **M1 M1** for a correct pair of factors; $x = \frac{1}{3}$ or $x = 2$ **A1**.

(c) $a = 1$, $b = 6$, $c = 4$, so $x = \frac{-6 \pm \sqrt{36 - 16}}{2}$ **M1 M1** for the substituted formula, for $\sqrt{20}$; $\sqrt{20} = 4.472$ **M1**; $x = -0.76$ or $x = -5.24$ **A1**.