Boolean Logic

IGCSE Computer Science ▪ Topic 10

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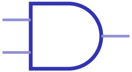
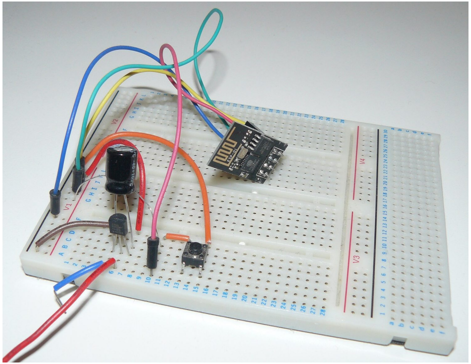


What we will cover

1 Gates

What is Boolean logic?

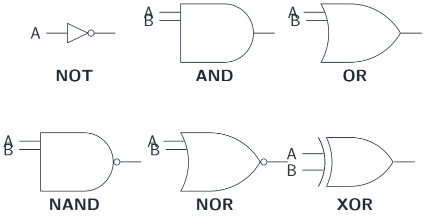
Boolean logic 布尔逻辑 works with just two values: TRUE (1) and FALSE (0). A **logic gate** 逻辑门 takes one or more inputs and gives one output. A **truth table** 真值表 lists the output for every combination of inputs.



Breadboard

The six logic gates

- **NOT** – invert
- **AND** – 1 only if all inputs are 1
- **OR** – 1 if any input is 1
- **NAND** / **NOR** – universal (any circuit from these)
- **XOR** – 1 if the inputs differ



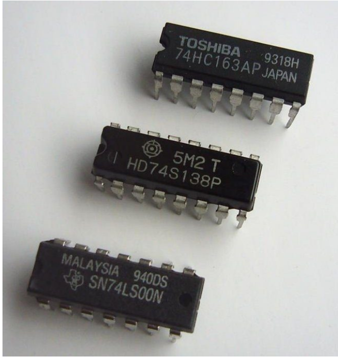
The six logic gates. A small circle on the output means the result is inverted (NOT, NAND, NOR)

The six gates, and the rule each one follows

NOT gate	One input. The **NOT gate** 非门 reverses it: output is 1 when the input is 0.
AND gate	Output 1 only when both inputs are 1.
OR gate	The **OR gate** 或门 gives 1 when at least one input is 1.
NAND gate	The **NAND gate** 与非门 is AND followed by NOT – the opposite of AND.
NOR gate	The **NOR gate** 或非门 is OR followed by NOT – the opposite of OR.
XOR gate	The **XOR gate** 异或门 (exclusive OR) gives 1 when the inputs are different.

A small circle on the output means the result is inverted – which is why NOT, NAND and NOR all carry one.

A real logic chip



A real logic chip: inside are logic gates like the ones on this page

Truth tables: AND and OR

A	B	$A \cdot B$
0	0	0
0	1	0
1	0	0
1	1	1

AND

AND: 1 only when both are 1

A	B	A+B
0	0	0
0	1	1
1	0	1
1	1	1

OR

> Build a truth table row by row: with n inputs there are 2^n rows – 2 inputs give 4, 3 inputs give 8. List them in binary-counting order so you miss none.

Three inputs give eight rows

3 inputs $\Rightarrow 2 \times 2 \times 2 =$ **8** rows

A	B	C
0	0	0
0	0	1
0	1	0
0	1	1
1	0	0
1	0	1
1	1	0
1	1	1

list **every** combination - count up in binary, from 000 to 111

the last column flips every row, the next every two rows, the first every four

Three inputs give eight rows: every combination counted in binary

2 Expressions and circuits

Logic expressions

A **logic expression** 逻辑表达式 writes a circuit in words:

$$X = (A \texttt{ AND } B) \texttt{ OR } (\texttt{NOT } C)$$

Work it out inside the brackets first, then combine.

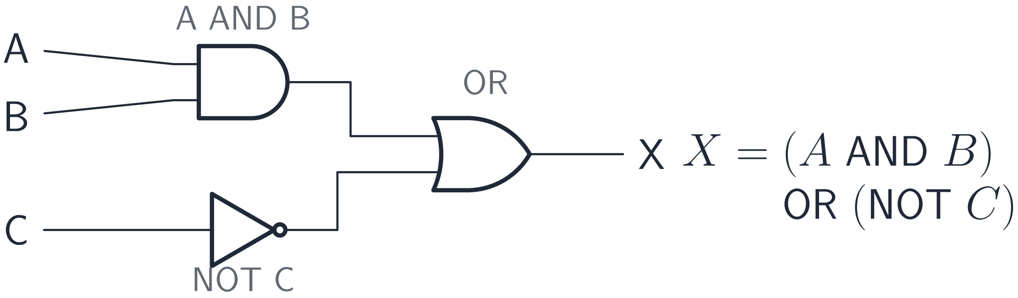
> Precedence: `NOT` binds tightest, then `AND`, then `OR`. Use brackets to be sure – an examiner reads exactly what you wrote.

Four ways to say the same logic

a problem statement	an alarm (X) sounds when the door is open (A) AND the system is switched on (B)
a logic expression	`X = A AND B`
a logic circuit	the gates drawn with their inputs and output
a truth table	every input combination and the resulting output
you must convert	between all four, in either direction
reading a statement	pick out the conditions, then the logic words – *and*, *or*, *not*

With n inputs a truth table has 2^n rows: 2 inputs give 4, 3 inputs give 8. Counting the rows first stops you leaving one out.

The expression X = (A AND B)



The expression X = (A AND B) OR (NOT C) drawn as a logic circuit

Worked example – one row of a truth table

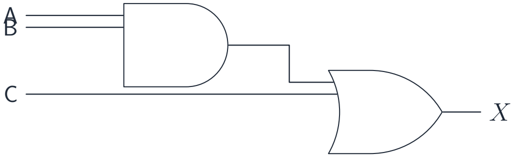
Complete `X = (A AND B) OR (NOT C)` for the row $A = 1$, $B = 0$, $C = 0$.

- work outwards from the brackets, one operator at a time
- $\texttt{A AND B} = 1 \text{ AND } 0 = 0$
- $\texttt{NOT C} = \text{NOT } 0 = 1$
- $\texttt{0 OR 1} = 1$

Add a column per bracket to the table rather than holding the intermediate values in your head. The marks are for those columns as much as for the answer.

Logic circuits

$$X = (A \textbf{ AND } B) \textbf{ OR } C$$



A **logic circuit** 逻辑电路 draws the gates joined by wires. To build its truth table, work left to right: find each gate's output, feed it into the next.

> Circuit $\to$ expression $\to$ truth table are three views of the same logic – an exam may give you any one and ask for another.

Exam tips

- Learn all six gates and their truth tables: **NOT**, **AND**, **OR**, **NAND** (NOT AND), **NOR** (NOT OR), **XOR** (output 1 when the inputs are different).
- Build a truth table with *all* input rows (2 inputs $\rightarrow$ 4 rows, 3 inputs $\rightarrow$ 8), counting up in binary, and add a working column for each gate.
- Turn a problem statement into a logic expression by picking out the AND / OR / NOT words, then draw it exactly as written – **do not simplify** it.
- NAND and NOR give the *opposite* output to AND and OR; a small circle on a gate's output means the result is inverted.

3 Recap

Topic 10 – key takeaways

1 Two values

TRUE (1) and FALSE (0), through gates

2 Six gates

NOT, AND, OR, NAND, NOR, XOR

3 Truth table

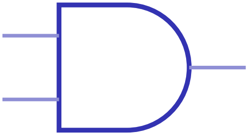
2^n rows for n inputs, in order

4 Three views

circuit, expression, table – same logic

Check yourself

1. When is an AND gate's output 1?
2. How many rows for 3 inputs?
3. Which gate outputs 1 when inputs differ?
4. Which operator binds tightest?



Answers **1.** when both inputs are 1 **2.** 8 **3.** XOR **4.** NOT