

10 Boolean logic

Name: _____ Class: _____ Date: _____

Total: 24 marks**KEY WORDS** logic gate • truth table • logic expression • AND • XOR

Objective

Build the skills to answer exam questions on **Boolean logic** : the logic gates NOT, AND, OR, NAND, NOR and XOR, logic expressions, and truth tables for circuits with up to three inputs and one output.

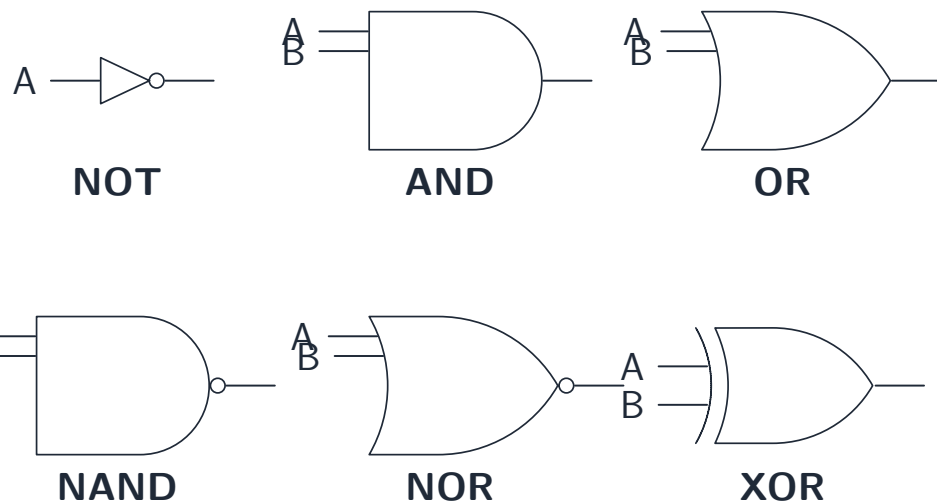
You must be able to:

- state the output of each gate
- write a logic expression from a short problem statement
- complete a truth table from an expression, without simplifying the expression

1 Key knowledge and worked examples

■ The six gates

A **logic gate** takes inputs that are **1** (true) or **0** (false) and gives one output. NOT has one input. The other gates have two inputs at IGCSE.



A small circle on the output means the result is inverted

Gate	Output is 1 when
NOT	the input is 0
AND	both inputs are 1
OR	at least one input is 1
NAND	AND would be 0 (NOT AND)
NOR	OR would be 0 (NOT OR)
XOR	the two inputs are different

AND outputs:

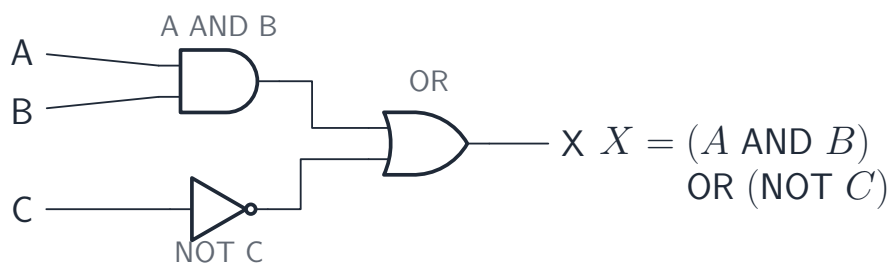
A	B	A AND B
0	0	0
0	1	0
1	0	0
1	1	1

NOR outputs (the opposite of OR):

A	B	A NOR B
0	0	1
0	1	0
1	0	0
1	1	0

■ Expressions and truth tables

A **logic expression** writes the gates in words, such as (A AND B) OR (NOT C). Draw or complete the table for that expression. Do not simplify it first.



Work one gate at a time: AND and NOT first, then OR

3 inputs $\Rightarrow 2 \times 2 \times 2 = 8$ rows

A	B	C
0	0	0
0	0	1
0	1	0
0	1	1
1	0	0
1	0	1
1	1	0
1	1	1

list **every** combination -
count up in binary,
from 000 to 111
the last column flips
every row, the next
every two rows,
the first every four

Two inputs need 4 rows; three inputs need 8 rows

Worked example. One row of $X = (A \text{ AND } B) \text{ OR } (\text{NOT } C)$, with $A = 1$, $B = 0$, $C = 0$.

1. $A \text{ AND } B = 1 \text{ AND } 0 = 0$, because AND needs both inputs to be 1.
2. $\text{NOT } C = \text{NOT } 0 = 1$.
3. $X = 0 \text{ OR } 1 = 1$.

Give each gate its own column. The final column is the output.

2 Practice

2.1 Give the output of NOT A when A is 0, and when A is 1. [2]

2.2 State the only input pair that makes an AND gate output 1. [1]

2.3 State when an XOR gate outputs 1. [1]

2.4 Explain what a NAND gate does, and give its output when both inputs are 1. [2]

2.5 Complete the outputs of $X = A \text{ NOR } B$. [4]

A	B	X
0	0	
0	1	
1	0	
1	1	

2.6 An alarm X sounds when the door is open (A) AND the system is on (B). Write the logic expression for X. [2]

3 Exam-style questions

3.1 The gate that outputs 1 only when both inputs are 1 is: [1]

- ☐ A OR
☐ B XOR
☐ C AND
☐ D NOR

3.2 Complete the truth table for $X = \text{NOT } (A \text{ OR } B)$. [4]

A	B	A OR B	X
0	0		
0	1		
1	0		
1	1		

3.3 A lamp X is on when switch A is on AND switch B is off.

(a) Write the logic expression for X. Do not simplify a different expression into this one.[2]

(b) Find X when $A = 1$ and $B = 0$. Show the output of each gate. [2]

3.4 $X = (A \text{ AND } B) \text{ OR } C$.

(a) State how many rows a full truth table for A, B and C must have. [1]

(b) Find X when $A = 1$, $B = 1$ and $C = 0$. Show the AND result, then the OR result.[2]

Solutions

Each answer shows the steps, then the **result**.

2.1

- NOT 0 = **1**
- NOT 1 = **0**

2.2

- both inputs are 1, written **1 and 1**

2.3

- the output is 1 when the inputs are **different**

2.4

- NAND is NOT AND: it outputs the opposite of AND
- when both inputs are 1, AND is 1, so NAND is **0**

2.5

- 0 NOR 0 = **1**
- 0 NOR 1 = **0**
- 1 NOR 0 = **0**
- 1 NOR 1 = **0**

2.6

- both conditions are required, so the gate is AND
- **X = A AND B**

3.1 C

- AND is 1 only for inputs 1 and 1
- OR is also 1 when one input is 1; XOR is 1 when the inputs differ; NOR is 1 only for 0 and 0

3.2

- OR column: **0, 1, 1, 1**
- X is NOT of that column: **1, 0, 0, 0**

A	B	A OR B	X
0	0	0	1
0	1	1	0
1	0	1	0
1	1	1	0

3.3

(a)

- switch B is off, so use NOT B
- both parts are required: $X = A \text{ AND } (\text{NOT } B)$

(b)

- $\text{NOT } B = \text{NOT } 0 = 1$
- $A \text{ AND } 1 = 1 \text{ AND } 1 = 1$, so $X = 1$

3.4

(a)

- three inputs give $2^3 = 8$ rows

(b)

- $A \text{ AND } B = 1 \text{ AND } 1 = 1$
- $1 \text{ OR } C = 1 \text{ OR } 0 = 1$, so $X = 1$