

## 5.2 Root finding and numerical integration

---

Name: \_\_\_\_\_ Class: \_\_\_\_\_ Date: \_\_\_\_\_

**Total: 13 marks**

**KEY WORDS** iteration

### Objective

---

Root finding and numerical integration. Supported SL concept preparation; original classroom practice, not an official IB paper or assessed learner work.

### 1 Key knowledge and worked examples

---

#### ■ How reliable is an approximation?

- A nonlinear equation has no convenient exact solution. An approximation still needs evidence that it is reliable.
  - This lesson studies **iteration** : A repeated update in which each new approximation is calculated from the previous one.
- 

#### ■ Choose the mathematical structure

- A continuous function with opposite signs at two endpoints has a root between them. Newton's method uses  $x_{\text{next}} = x - \frac{f(x)}{f'(x)}$ , with a nonzero derivative. The trapezium rule approximates a definite integral using endpoint heights.
  - State the allowed inputs and units before calculating. An equation should express the relationship, not just record a calculator entry.
- 

#### ■ Work through a checked case

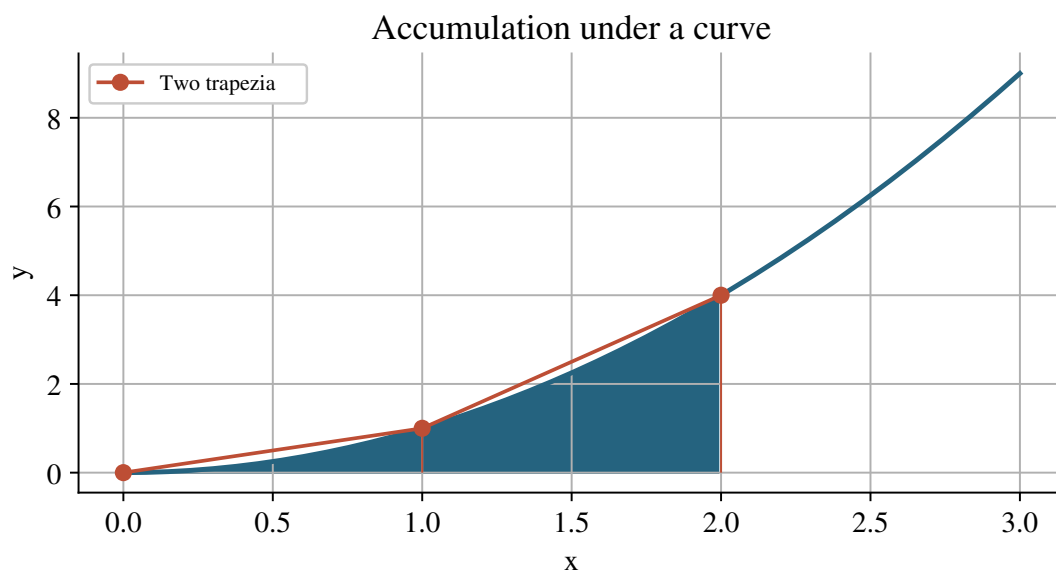
$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

- Check the result against the starting quantities. Substitute into the original relation, or compare the graph and numerical answer where appropriate.

#### Example:

For  $f(x) = x^2 - 2$  and  $x_0 = 1.5$ , Newton gives  $x_1 = 1.5 - (2.25 - 2)/3 = 1.4166667$ . With  $y = x^2$  on  $[0, 2]$  and two equal strips,  $h = 1$  and trapezium area  $= (1/2)(0 + 2 \times 1 + 4) = 3$ ; the exact area

is  $8/3$ .



---

### ■ Test a tempting shortcut

- A sign change across a discontinuity does not prove a root. Iteration can diverge or cycle. The trapezium rule's overestimate or underestimate depends on curvature, not just whether the function increases.
- When a shortcut fails, identify the assumption it breaks. Keep an exact value until the requested final rounding.

### Warn:

Every sign change proves a root, including one across a discontinuity. This claim is false. Explain which definition or assumption it violates.

---

### ■ Interpret a new situation

- Give a stopping criterion and report sensible accuracy. Confirm the approximated root by a sign bracket around the stated rounded answer; explain any failure of the chosen iteration.
  - A complete solution gives the mathematical result and explains what it means. Check that it is possible in the stated context.
- 

### ■ Use this in your course

- Current first-assessment-2021 Applications and Interpretation SL. This is authored concept support; the full guide is needed to certify every objective.
- Give the method before the final answer, and use the paper's calculator and formula rules. Review a wrong answer by locating the first invalid step.

### Key:

A repeated update in which each new approximation is calculated from the previous one. Choose the relationship, show the method, check its assumptions and interpret the result.

## 2 Practice

---

**2.1** After one Newton step for  $x^2-2$  from  $x_0=1.5$ , find  $x_1$ . Show the relation, substitution and units. [2]

---

---

---

**2.2** Use two strips for the trapezium estimate of integral  $x^2$  from 0 to 2. Show the relation, substitution and units. [2]

---

---

---

**2.3** Find the exact integral of  $x^2$  from 0 to 2. Show the relation, substitution and units.[2]

---

---

---

**2.4** Decide whether this claim is true and justify: Every sign change proves a root, including one across a discontinuity. [2]

---

---

---

**2.5** Which description correctly defines iteration? Choose one answer.

- **A** A repeated update in which each new approximation is calculated from the previous one.
- **B** A numerical answer that needs no assumptions or interpretation
- **C** A rule that remains valid even when its required conditions fail [1]

---

### 3 Exam-style questions

---

**3.1** Apply one Newton step to  $f(x)=x^3-2x+2$  from  $x_0=0$  and then one more. What happens, and what does it teach about convergence? [4]

---

---

---

---

---

---

---

---

---

---

## Solutions

---

**2.1**  $x_1 = 1.5 - (1.5^2 - 2)/(2 \times 1.5) = 17/12 \approx 1.416667$ .

**2.2**  $h=1$  with heights 0,1,4 gives  $(1/2)(0+2 \times 1+4)=3$ .

**2.3** The exact integral is  $[x^3/3]$  from 0 to 2  $= 8/3$ .

**2.4** False. A sign change across a discontinuity does not prove a root. Iteration can diverge or cycle. The trapezium rule's overestimate or underestimate depends on curvature, not just whether the function increases.

**2.5 A** A repeated update in which each new approximation is calculated from the previous one.

**3.1**  $x_1 = 0 - 2/(-2) = 1$ ;  $x_2 = 1 - 1/1 = 0$ . It cycles 0, 1 rather than converging. Newton requires suitable starting values/derivatives; use a bracketed method or inspect the function rather than assuming every iteration converges.