

Number and algebra · supported focus

Supported focus · HL

Supported scope

- Supported HL focus. First assessment 2021; current through 2028.
First-assessment-2029 course is separate.. Remaining guide, assessment and practical requirements retain their recorded holds.
- State model conditions and distinguish fictional data from measurements.

Number · 1.1

Reasoning

A supplier packs 72 pencils and 90 pens into identical gift bags. How can we avoid leftovers? This lesson studies prime factor : A prime number that divides the integer exactly.

Number · 2.1

Reasoning

Prime factors reveal shared structure. Use the smallest common prime powers for the HCF and the largest for the LCM. Estimate before calculating; use brackets to preserve the order of operations. State the allowed inputs and units before calculating. An equation should express the relationship, not just record a calculator entry.

Number · 3.1

Reasoning

Use the displayed relationship within its stated domain.

$$a = \prod p_i^{\alpha_i}, \quad b = \prod p_i^{\beta_i}, \quad \text{HCF}(a, b) = \prod p_i^{\min(\alpha_i, \beta_i)}$$

Number · 3.2

Reasoning

Check the result against the starting quantities. Substitute into the original relation, or compare the graph and numerical answer where appropriate.

Number · 3.3

Reasoning

$72 = 2^3 \times 3^2$ and $90 = 2 \times 3^2 \times 5$. Their HCF is $2 \times 9 = 18$. Make 18 bags with 4 pencils and 5 pens each. Their LCM is $2^3 \times 3^2 \times 5 = 360$.

Number · 4.1

Reasoning

The HCF divides both numbers; the LCM is a multiple of both. They answer different questions. A decimal estimate is not an exact fraction. When a shortcut fails, identify the assumption it breaks. Keep an exact value until the requested final rounding.

Number · 4.2

Reasoning

The HCF of two positive integers is always larger than either integer. This claim is false. Explain which definition or assumption it violates.

Number · 5.1

Reasoning

For a non-calculator paper, keep fractions exact and show cancellation. For a calculator paper, enter the full expression and compare with your estimate. A complete solution gives the mathematical result and explains what it means. Check that it is possible in the stated context.

Number · 6.1

Reasoning

Current first-assessment-2021 Applications and Interpretation HL. This is authored concept support; the full guide is needed to certify every objective. Give the method before the final answer, and use the paper's calculator and formula rules. Review a wrong answer by locating the first invalid step.

Number · 6.2

Reasoning

A prime number that divides the integer exactly. Choose the relationship, show the method, check its assumptions and interpret the result.

Number · diagram

$$72 = 2^3 \times 3^2$$

$$90 = 2 \times 3^2 \times 5$$

$$\text{Shared smallest powers: } 2 \times 3^2 = 18$$

- Original teaching diagram: Exact arithmetic and estimation
- Use the labels and the stated model together.

Number · original transfer

Your turn

A supplier has 84 pencils and 126 pens. Find the greatest number of identical bags without leftovers and the contents of each; then explain why the LCM answers a different question.

Number · transfer reasoning

Method and interpretation

HCF=42, each bag 2 pencils and 3 pens. LCM=252 is a common multiple, useful for matching repeated quantities, not the greatest number of bags dividing both stocks.

Percent · 1.1

Reasoning

A coat is reduced by 20% to ¥240. The discount applies to the original price, not to the sale price. This lesson studies multiplier : A factor that performs a percentage change in one multiplication.

Percent · 2.1

Reasoning

A $p\%$ increase has multiplier $1+p/100$; a decrease has multiplier $1-p/100$. Reverse a percentage by dividing by the multiplier. In a ratio, first find the total number of parts. State the allowed inputs and units before calculating. An equation should express the relationship, not just record a calculator entry.

Percent · 3.1

Reasoning

Use the displayed relationship within its stated domain.

$$P_{\text{new}} = P_{\text{old}} \left(1 + \frac{r}{100} \right)$$

Percent · 3.2

Reasoning

Check the result against the starting quantities. Substitute into the original relation, or compare the graph and numerical answer where appropriate.

Percent · 3.3

Reasoning

Let the original price be P . The model is sale price $= 0.8P$. Hence $P = 240 / 0.8 = 300$. A later 20% increase gives $240 \times 1.2 = 288$, so the two changes do not cancel.

Percent · 4.1

Reasoning

A percentage uses a stated base. Subtracting the percentages loses that base. For compound change, multiply the multipliers; do not add the percentages. When a shortcut fails, identify the assumption it breaks. Keep an exact value until the requested final rounding.

Percent · 4.2

Reasoning

A 20% decrease followed by a 20% increase restores the starting price. This claim is false. Explain which definition or assumption it violates.

Percent · 5.1

Reasoning

For direct proportion use $y=kx$; for inverse proportion use $y=k/x$. Calculate k from a known pair before using a new value. State what you held constant. A complete solution gives the mathematical result and explains what it means. Check that it is possible in the stated context.

Percent · 6.1

Reasoning

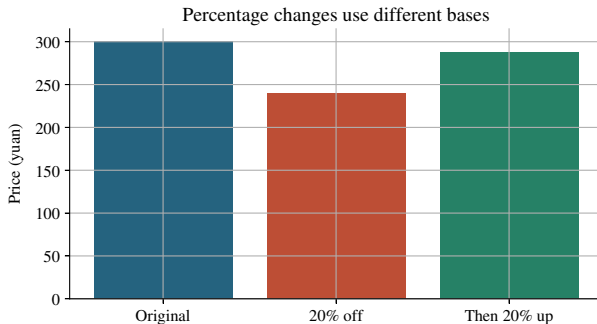
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Percent · 6.2

Reasoning

A factor that performs a percentage change in one multiplication. Choose the relationship, show the method, check its assumptions and interpret the result.

Percent · diagram



- Original teaching diagram: Percentages, ratio and proportional reasoning
- Use the labels and the stated model together.

Percent · original transfer

Your turn

A price rises 20% then falls 20%. Find the net change and the percentage increase needed to recover the original after that pair.

Percent · transfer reasoning

Method and interpretation

Factor $1.2 \times 0.8 = 0.96$, a 4% fall. Recovery factor $1/0.96$, giving 4.1667% increase. Both percentage changes use different bases.

Sequences · 1.1

Reasoning

A saving plan adds ¥30 more each week; a population model grows by 5% each year. Equal differences and equal ratios need different models. This lesson studies common ratio : The constant multiplier between consecutive terms of a geometric sequence.

Sequences · 2.1

Reasoning

For an arithmetic progression, $u_n = a + (n-1)d$ and $S_n = n(2a + (n-1)d)/2$. For a geometric progression, $u_n = ar^{(n-1)}$ and $S_n = a(1-r^n)/(1-r)$. An infinite geometric sum exists only if $|r| < 1$. State the allowed inputs and units before calculating. An equation should express the relationship, not just record a calculator entry.

Sequences · 3.1

Reasoning

Use the displayed relationship within its stated domain.

$$u_n = a + (n - 1)d, \quad S_n = \frac{n}{2} [2a + (n - 1)d]$$

Sequences · 3.2

Reasoning

Check the result against the starting quantities. Substitute into the original relation, or compare the graph and numerical answer where appropriate.

Sequences · 3.3

Reasoning

For $a=5, d=3, n=8$, $u_8=5+7 \times 3=26$ and $S_8=8(10+21)/2=124$. For $a=12, r=1/2$, $S_{\text{infinity}}=12/(1-1/2)=24$. For $u_{(n+1)}=2u_n+1$ with $u_1=1$, the next terms are 3, 7, 15.

Sequences · 4.1

Reasoning

The first term has index 1, so the exponent is $n-1$. A sequence is a list; a series is a sum. A geometric sequence can alternate in sign and still converge. When a shortcut fails, identify the assumption it breaks. Keep an exact value until the requested final rounding.

Sequences · 4.2

Reasoning

Every geometric series has a finite sum to infinity. This claim is false. Explain which definition or assumption it violates.

Sequences · 5.1

Reasoning

Explain whether the context justifies additive or multiplicative change. In finance, distinguish a single deposit from a stream of deposits before choosing a sum formula. A complete solution gives the mathematical result and explains what it means. Check that it is possible in the stated context.

Sequences · 6.1

Reasoning

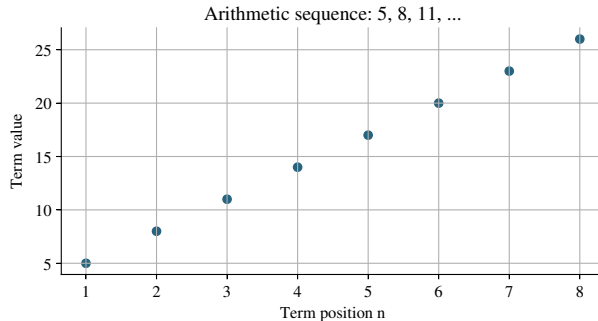
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Sequences · 6.2

Reasoning

The constant multiplier between consecutive terms of a geometric sequence. Choose the relationship, show the method, check its assumptions and interpret the result.

Sequences · diagram



- Original teaching diagram: Sequences, series and recurrence
- Use the labels and the stated model together.

Sequences · original transfer

Your turn

A sequence obeys $u_1=4$ and $u_{n+1}=0.5 u_n+3$. Find its fixed point and express u_n-6 as a geometric sequence. Does the sum of u_n converge?

Sequences · transfer reasoning

Method and interpretation

Fixed point 6. Subtracting 6 gives $u_{n+1}-6=0.5(u_n-6)$, so $u_n=6-2(0.5)^{n-1}$.

Terms approach 6, so their infinite sum diverges; a convergent sequence need not give a convergent series.

Modelling · 1.1

Reasoning

A line predicts bus travel time from distance. A small calculation error matters less than using a model outside its evidence. This lesson studies residual : The observed value minus the value predicted by a fitted model.

Modelling · 2.1

Reasoning

A linear model $y=a+bx$ has intercept a and slope b with contextual units. Inspect residuals and the data range. For repeated percentage change use a geometric model; for a loan distinguish principal, rate, repayment and period. State the allowed inputs and units before calculating. An equation should express the relationship, not just record a calculator entry.

Modelling · 3.1

Reasoning

Use the displayed relationship within its stated domain.

$$\hat{t} = 8 + 2d, \quad e = t - \hat{t}$$

Modelling · 3.2

Reasoning

Check the result against the starting quantities. Substitute into the original relation, or compare the graph and numerical answer where appropriate.

Modelling · 3.3

Reasoning

The fitted model $t=8+2d$ predicts 18 minutes at $d=5$ km. If the observed time is 21, the residual is 3 minutes. A ¥1000 deposit at 5% compound annual interest becomes $1000 \times 1.05^3 = 1157.625$ after 3 years.

Modelling · 4.1

Reasoning

A good fit does not prove a causal mechanism. A correlation coefficient measures linear association, not the gradient. Calculator output must be translated into a model, checked and interpreted. When a shortcut fails, identify the assumption it breaks. Keep an exact value until the requested final rounding.

Modelling · 4.2

Reasoning

A high correlation permits reliable extrapolation to any distance. This claim is false. Explain which definition or assumption it violates.

Modelling · 5.1

Reasoning

In IB AI, record the data source, domain, assumptions and calculator method. Compare an alternative model and judge predictions against residuals. A financial answer must state payment timing and whether interest is compounded. A complete solution gives the mathematical result and explains what it means. Check that it is possible in the stated context.

Modelling · 6.1

Reasoning

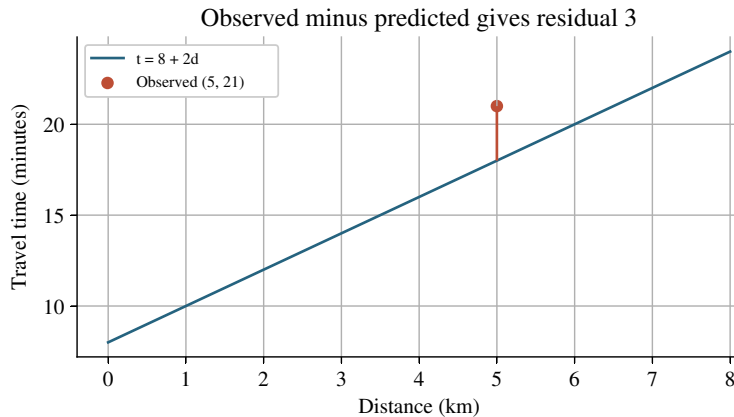
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Modelling · 6.2

Reasoning

The observed value minus the value predicted by a fitted model. Choose the relationship, show the method, check its assumptions and interpret the result.

Modelling · diagram



Residual = observed value minus model prediction.

Modelling · original transfer

Your turn

A fictional journey model is $t=8+2d$. An observed trip at $d=5$ takes 21 minutes. Find the residual, predict at $d=50$ and explain why that prediction needs caution.

Modelling · transfer reasoning

Method and interpretation

Prediction 18, residual+3 minutes; at 50 prediction 108. If the fitted distance range was short, this is extrapolation; traffic, stops and model form may change. Regression does not establish a causal coefficient by itself.

Matrices · 1.1

Reasoning

A map stretches one direction and shears another. A matrix records the transformation of each basis vector. This lesson studies determinant : A scalar that determines whether a square matrix is invertible and, in two dimensions, its signed area factor.

Matrices · 2.1

Reasoning

A 2×2 matrix $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ has determinant $ad-bc$ and inverse $\begin{bmatrix} d & -b \\ -c & a \end{bmatrix} / (ad-bc)$ if the determinant is nonzero. Transform column vectors by left multiplication. The rightmost matrix acts first in a product. State the allowed inputs and units before calculating. An equation should express the relationship, not just record a calculator entry.

Matrices · 3.1

Reasoning

Use the displayed relationship within its stated domain.

$$\det \begin{pmatrix} a & b \\ c & d \end{pmatrix} = ad - bc$$

Matrices · 3.2

Reasoning

Check the result against the starting quantities. Substitute into the original relation, or compare the graph and numerical answer where appropriate.

Matrices · 3.3

Reasoning

For $A = \begin{bmatrix} 2 & 1 \\ 0 & 3 \end{bmatrix}$, $\det A = 6$. $A(1,2) = (4,6)$. Its inverse is $\begin{bmatrix} 1/2 & -1/6 \\ 0 & 1/3 \end{bmatrix}$. The characteristic equation is $(2-\lambda)(3-\lambda)=0$, so the eigenvalues are 2 and 3.

Matrices · 4.1

Reasoning

Matrix multiplication is usually not commutative. A determinant of zero means the transformation loses a dimension and has no inverse. Eigenvalues and diagonalisation are advanced scope, not ordinary GCSE vector content. When a shortcut fails, identify the assumption it breaks. Keep an exact value until the requested final rounding.

Matrices · 4.2

Reasoning

Every square matrix has an inverse. This claim is false. Explain which definition or assumption it violates.

Matrices · 5.1

Reasoning

Find an eigenvector by solving $(A - \lambda I)v = 0$ with $v \neq 0$. Explain the geometrical meaning: this vector keeps its line direction under the transformation. A complete solution gives the mathematical result and explains what it means. Check that it is possible in the stated context.

Matrices · 6.1

Reasoning

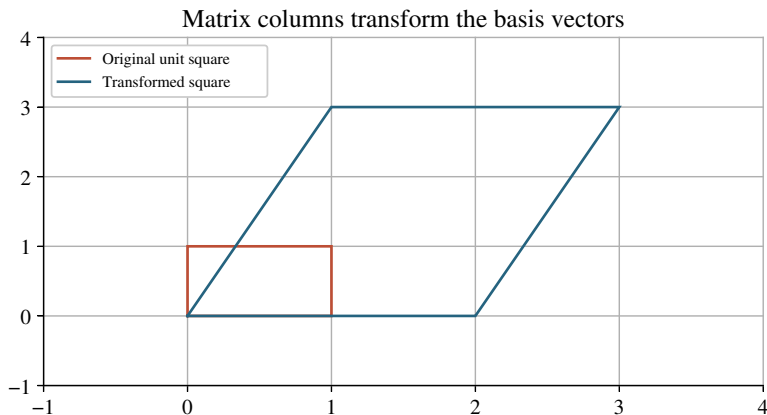
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Matrices · 6.2

Reasoning

A scalar that determines whether a square matrix is invertible and, in two dimensions, its signed area factor. Choose the relationship, show the method, check its assumptions and interpret the result.

Matrices · diagram



Matrix columns are the transformed basis vectors.

Matrices · original transfer

Your turn

For $A = \begin{bmatrix} 2 & 1 \\ 0 & 3 \end{bmatrix}$, find an eigenvector for each eigenvalue and check whether $(1, 1)$ is an eigenvector.

Matrices · transfer reasoning

Method and interpretation

λ_2 :eigenvector(1, 0); λ_3 :(1, 1). $A(1, 1)=(3, 3)=3(1, 1)$. Nonzero scalar multiples also work; the zero vector is excluded.

Complex · 1.1

Reasoning

The equation $x^2+1=0$ has no real roots. Extending the number system lets us represent and calculate with both roots. This lesson studies complex conjugate : The number obtained by reversing the sign of the imaginary part.

Complex · 2.1

Reasoning

Write $z=a+bi$ with $i^2=-1$. The conjugate is $a-bi$ and z times its conjugate $=a^2+b^2$. The modulus is $\sqrt{a^2+b^2}$; the argument needs the correct quadrant. Conjugate roots occur for polynomials with real coefficients. State the allowed inputs and units before calculating. An equation should express the relationship, not just record a calculator entry.

Complex · 3.1

Reasoning

Use the displayed relationship within its stated domain.

$$z = a + bi, \quad |z| = \sqrt{a^2 + b^2}, \quad z\bar{z} = a^2 + b^2$$

Complex · 3.2

Reasoning

Check the result against the starting quantities. Substitute into the original relation, or compare the graph and numerical answer where appropriate.

Complex · 3.3

Reasoning

For $z=3+4i$, $|z|=5$ and z conjugate $=3-4i$. The reciprocal is $(3-4i)/25$. For $x^2-6x+13=0$, the roots are $3\pm 2i$. Multiplying by the conjugate makes a complex denominator real.

Complex · 4.1

Reasoning

The square root of a sum is not the sum of the square roots. An inverse tangent alone may return the wrong quadrant. Do not confuse modulus with squared modulus. When a shortcut fails, identify the assumption it breaks. Keep an exact value until the requested final rounding.

Complex · 4.2

Reasoning

For $a+bi$, its modulus is always $a+b$. This claim is false. Explain which definition or assumption it violates.

Complex · 5.1

Reasoning

Draw the real axis horizontally and imaginary axis vertically. For products, moduli multiply and arguments add; powers extend this geometric pattern through de Moivre's theorem. A complete solution gives the mathematical result and explains what it means. Check that it is possible in the stated context.

Complex · 6.1

Reasoning

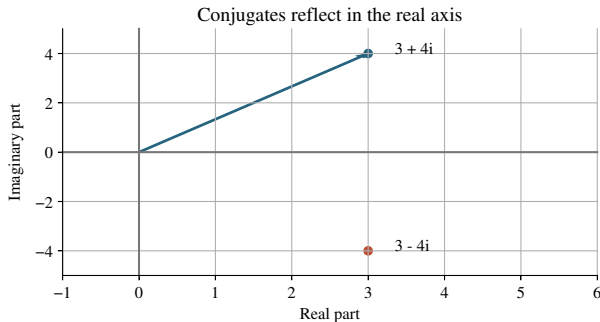
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Complex · 6.2

Reasoning

The number obtained by reversing the sign of the imaginary part. Choose the relationship, show the method, check its assumptions and interpret the result.

Complex · diagram



- Original teaching diagram: Complex numbers and roots
- Use the labels and the stated model together.

Complex · original transfer

Your turn

Find all complex roots of $z^2 = -8i$ in Cartesian form and verify them.

Complex · transfer reasoning

Method and interpretation

Polar modulus 8 and argument $-\pi/2$ give roots modulus $\sqrt{8}$, arguments $-\pi/4$ and $3\pi/4$: $2-2i$ and $-2+2i$. Squaring each gives $-8i$. Both roots are needed.

Check yourself

- Explain prime factor using the stated model.
- Explain multiplier using the stated model.
- Change one value in the original transfer; explain which assumption still matters.