

4.4 Hypothesis testing and contextual conclusions

Name: _____ Class: _____ Date: _____ **Total: 13 marks**

KEY WORDS significance level

Objective

Hypothesis testing and contextual conclusions. Supported HL concept preparation; original classroom practice, not an official IB paper or assessed learner work.

1 Key knowledge and worked examples

■ Could chance explain the result?

- A factory claims that only 10% of items are defective. A sample contains more defects, but chance alone may explain some difference.
- This lesson studies **significance level** : The chosen probability threshold for rejecting a null hypothesis.

■ Choose the mathematical structure

- State H_0 and H_1 in population parameters before inspecting the outcome. Calculate the appropriate tail probability under H_0 . Reject H_0 when the evidence meets the specified significance rule; otherwise say there is insufficient evidence.
- State the allowed inputs and units before calculating. An equation should express the relationship, not just record a calculator entry.

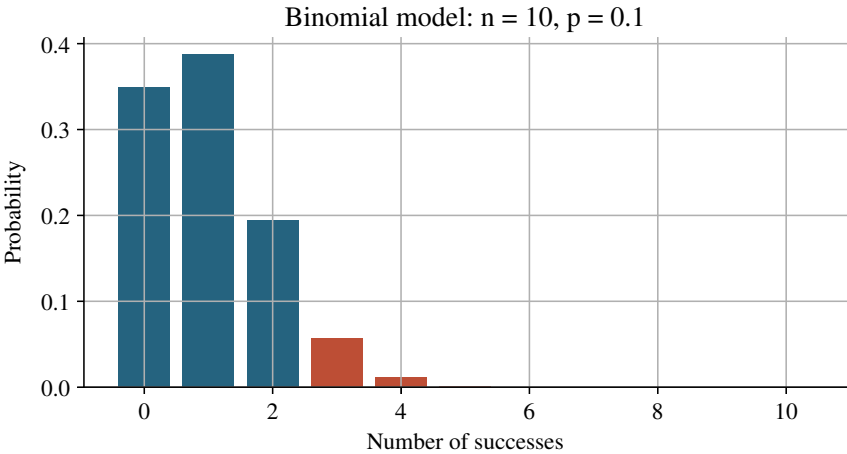
■ Work through a checked case

$$H_0 : p = p_0, \quad H_1 : p > p_0$$

- Check the result against the starting quantities. Substitute into the original relation, or compare the graph and numerical answer where appropriate.

Example:

For $H_0:p=0.1$ and $H_1:p>0.1$ with $n=10$, observing at least 3 defects has probability $1-(0.9^{10}+10\times0.1\times0.9^9+45\times0.1^2\times0.9^8)\approx0.070191$. At 5%, this is insufficient evidence that the defect rate exceeds 10%.



■ Test a tempting shortcut

- Failing to reject H_0 is not proof that H_0 is true. Choose the tail from H_1 , not from whichever tail gives a small result. Statistical significance does not measure the practical size of an effect.
- When a shortcut fails, identify the assumption it breaks. Keep an exact value until the requested final rounding.

Warn:

Failing to reject a null hypothesis proves it is true. This claim is false. Explain which definition or assumption it violates.

■ Interpret a new situation

- Finish with a sentence about the population and the original claim. State the model's assumptions and consider whether the sampling procedure supports them.
- A complete solution gives the mathematical result and explains what it means. Check that it is possible in the stated context.

■ Use this in your course

- Current first-assessment-2021 Applications and Interpretation HL. This is authored concept support; the full guide is needed to certify every objective.
- Give the method before the final answer, and use the paper's calculator and formula rules. Review a wrong answer by locating the first invalid step.

Key:

The chosen probability threshold for rejecting a null hypothesis. Choose the relationship, show the method, check its assumptions and interpret the result.

2 Practice

2.1 Find the significance threshold written as a decimal for 5%. Show the relation, substitution and units. [2]

2.2 A correct one-tailed p-value is 0.03 at a 5% threshold. Enter 1 for reject or 0 for do not reject. Show the relation, substitution and units. [2]

2.3 Find $P(X \geq 3)$ for X binomial(10,0.1). Show the relation, substitution and units. [2]

2.4 Decide whether this claim is true and justify: Failing to reject a null hypothesis proves it is true. [2]

2.5 Which description correctly defines significance level? Choose one answer.

- A The chosen probability threshold for rejecting a null hypothesis.
 - B A numerical answer that needs no assumptions or interpretation
 - C A rule that remains valid even when its required conditions fail
- [1]

3 Exam-style questions

3.1 A correctly designed test gives $p=0.04$. State decisions at 5% and 1% significance and explain why p is not the probability H_0 is true. [4]

Solutions

2.1 5% means $5/100=0.05$.

2.2 The correct tail probability 0.03 is below 0.05, so reject H_0 . This is evidence, not proof.

2.3 Sum the binomial probabilities for $k=3$ through 10; equivalently use $1-P(X \leq 2)=0.0701908264$.

2.4 False. Failing to reject H_0 is not proof that H_0 is true. Choose the tail from H_1 , not from whichever tail gives a small result. Statistical significance does not measure the practical size of an effect.

2.5 A The chosen probability threshold for rejecting a null hypothesis.

3.1 Reject at 5%, do not reject at 1%. p is a tail probability under H_0 and the model, not a posterior probability. Do not change the chosen threshold after seeing the data.