

2.1 Domains, inverses and composition

Name: _____ Class: _____ Date: _____ **Total: 13 marks**

KEY WORDS domain

Objective

Domains, inverses and composition. Supported HL concept preparation; original classroom practice, not an official IB paper or assessed learner work.

1 Key knowledge and worked examples

■ Which inputs are allowed?

- A square-root model returns a real output only for some inputs. Its formula alone does not specify a complete function.
- This lesson studies **domain** : The set of allowed inputs to a function.

■ Choose the mathematical structure

- State the domain and range. For an inverse, first ensure the function is one-to-one on its domain. Composition fg means apply g first, then f ; the intermediate output must be an allowed input to f .
- State the allowed inputs and units before calculating. An equation should express the relationship, not just record a calculator entry.

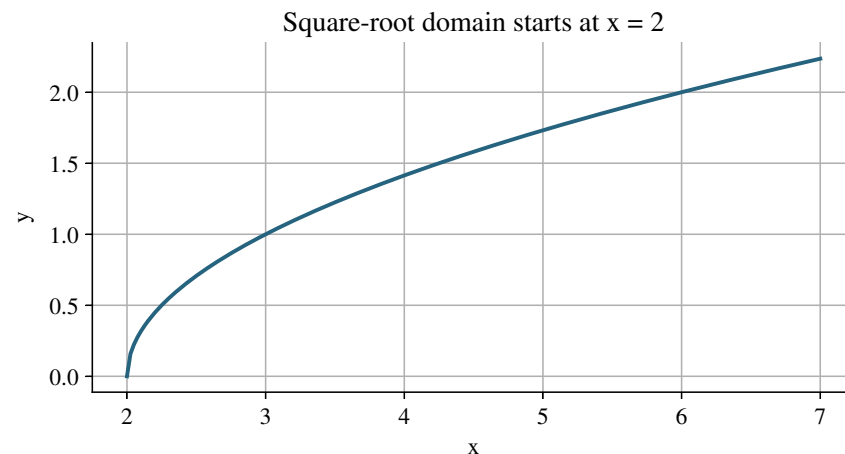
■ Work through a checked case

$$f(g(x)) = (f \circ g)(x), \quad f^{-1}(f(x)) = x$$

- Check the result against the starting quantities. Substitute into the original relation, or compare the graph and numerical answer where appropriate.

Example:

For $f(x) = \sqrt{x-2}$, $x \geq 2$ and the range is $y \geq 0$. From $y = \sqrt{x-2}$, $x = y^2 + 2$. Thus $f^{-1}(x) = x^2 + 2$ with $x \geq 0$. For $g(x) = x + 3$, $fg(1) = f(4) = \sqrt{2}$.



■ Test a tempting shortcut

- Squaring can introduce extraneous solutions. Restricting a parabola's domain is essential before claiming an inverse. A horizontal translation inside f has the opposite sign to the graph's movement.
- When a shortcut fails, identify the assumption it breaks. Keep an exact value until the requested final rounding.

Warn:

Every quadratic function on all real numbers has an inverse function. This claim is false. Explain which definition or assumption it violates.

■ Interpret a new situation

- Check $f(f^{-1}(x)) = x$ on the inverse domain. Use a sketch to test whether a horizontal line meets the original graph more than once.
- A complete solution gives the mathematical result and explains what it means. Check that it is possible in the stated context.

■ Use this in your course

- Current first-assessment-2021 Applications and Interpretation HL. This is authored concept support; the full guide is needed to certify every objective.
- Give the method before the final answer, and use the paper's calculator and formula rules. Review a wrong answer by locating the first invalid step.

Key:

The set of allowed inputs to a function. Choose the relationship, show the method, check its assumptions and interpret the result.

2 Practice

2.1 For $f(x)=2x+3$, find $f^{-1}(11)$. Show the relation, substitution and units. [2]

2.2 For $f(x)=2x+3$ and $g(x)=x^2$, find $fg(4)$. Show the relation, substitution and units.[2]

2.3 For $f(x)=\sqrt{x-2}$, find $f(11)$. Show the relation, substitution and units. [2]

2.4 Decide whether this claim is true and justify: Every quadratic function on all real numbers has an inverse function. [2]

2.5 Which description correctly defines domain? Choose one answer.

- **A** The set of allowed inputs to a function.
 - **B** A numerical answer that needs no assumptions or interpretation
 - **C** A rule that remains valid even when its required conditions fail
- [1]

3 Exam-style questions

3.1 For $f(x)=x^2$ on $x \geq 0$ and $g(x)=x-3$, find $f(g(x))$ and its domain, then find $f^{-1}(16)$. Explain why the restriction matters. [4]

Solutions

2.1 Solve $2x+3=11$, giving $x=4$.

2.2 Apply g first: $g(4)=16$; then $f(16)=2 \times 16+3=35$.

2.3 Substitute into the square root: $\sqrt{(11-2)}=\sqrt{9}=3$.

2.4 False. Squaring can introduce extraneous solutions. Restricting a parabola's domain is essential before claiming an inverse. A horizontal translation inside f has the opposite sign to the graph's movement.

2.5 A The set of allowed inputs to a function.

3.1 Composite $= (x-3)^2$ with $x \geq 3$ because $g(x)$ must lie in f 's domain. Inverse $16=4$. Without restriction x^2 is not one-to-one and the inverse would not be a function on the full real domain.