

1.5 Matrix transformations, inverses and eigenvalues

Name: _____ Class: _____ Date: _____

Total: 13 marks

KEY WORDS determinant

Objective

Matrix transformations, inverses and eigenvalues. Supported HL concept preparation; original classroom practice, not an official IB paper or assessed learner work.

1 Key knowledge and worked examples

■ Where do the basis vectors go?

- A map stretches one direction and shears another. A matrix records the transformation of each basis vector.
 - This lesson studies **determinant** : A scalar that determines whether a square matrix is invertible and, in two dimensions, its signed area factor.
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■ Choose the mathematical structure

- A 2×2 matrix $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ has determinant $ad-bc$ and inverse $\begin{bmatrix} d & -b \\ -c & a \end{bmatrix} / (ad-bc)$ if the determinant is nonzero. Transform column vectors by left multiplication. The rightmost matrix acts first in a product.
 - State the allowed inputs and units before calculating. An equation should express the relationship, not just record a calculator entry.
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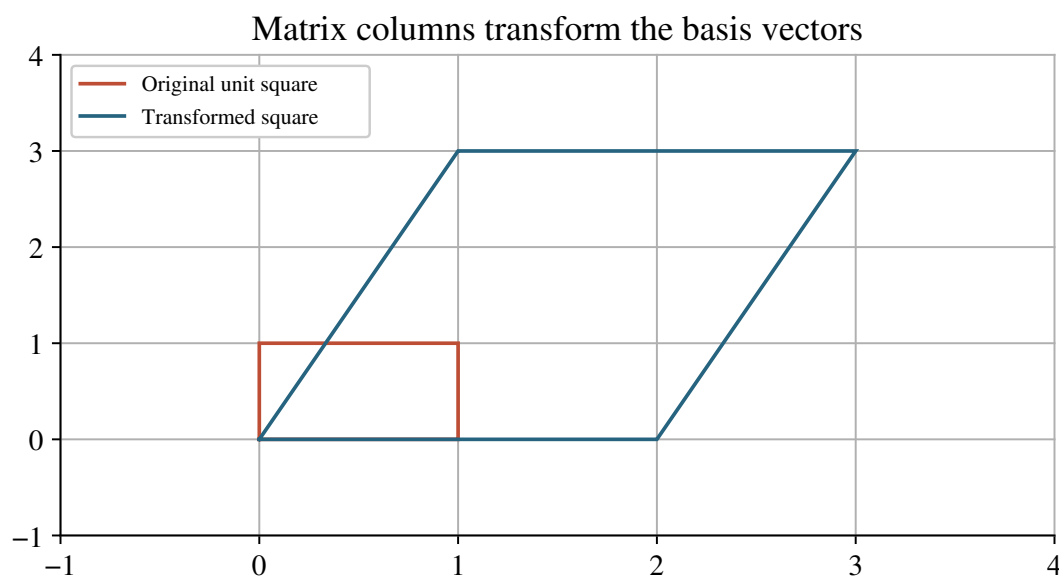
■ Work through a checked case

$$\det \begin{pmatrix} a & b \\ c & d \end{pmatrix} = ad - bc$$

- Check the result against the starting quantities. Substitute into the original relation, or compare the graph and numerical answer where appropriate.

Example:

For $A = \begin{bmatrix} 2 & 1 \\ 0 & 3 \end{bmatrix}$, $\det A = 6$. $A(1,2) = (4,6)$. Its inverse is $\begin{bmatrix} 1/2 & -1/6 \\ 0 & 1/3 \end{bmatrix}$. The characteristic equation is $(2-\lambda)(3-\lambda)=0$, so the eigenvalues are 2 and 3.



■ Test a tempting shortcut

- Matrix multiplication is usually not commutative. A determinant of zero means the transformation loses a dimension and has no inverse. Eigenvalues and diagonalisation are advanced scope, not ordinary GCSE vector content.
- When a shortcut fails, identify the assumption it breaks. Keep an exact value until the requested final rounding.

Warn:

Every square matrix has an inverse. This claim is false. Explain which definition or assumption it violates.

■ Interpret a new situation

- Find an eigenvector by solving $(A - \lambda I)v = 0$ with $v \neq 0$. Explain the geometrical meaning: this vector keeps its line direction under the transformation.
- A complete solution gives the mathematical result and explains what it means. Check that it is possible in the stated context.

■ Use this in your course

- Current first-assessment-2021 Applications and Interpretation HL. This is authored concept support; the full guide is needed to certify every objective.
- Give the method before the final answer, and use the paper's calculator and formula rules. Review a wrong answer by locating the first invalid step.

Key:

A scalar that determines whether a square matrix is invertible and, in two dimensions, its signed area factor. Choose the relationship, show the method, check its assumptions

and interpret the result.

2 Practice

2.1 Find the determinant of $\begin{bmatrix} 2 & 1 \\ 0 & 3 \end{bmatrix}$. Show the relation, substitution and units. [2]

2.2 Find the x-component after that matrix transforms (1,2). Show the relation, substitution and units. [2]

2.3 Find the larger eigenvalue of that matrix. Show the relation, substitution and units. [2]

2.4 Decide whether this claim is true and justify: Every square matrix has an inverse. [2]

2.5 Which description correctly defines determinant? Choose one answer.

- **A** A scalar that determines whether a square matrix is invertible and, in two dimensions, its signed area factor.
- **B** A numerical answer that needs no assumptions or interpretation
- **C** A rule that remains valid even when its required conditions fail [1]

3 Exam-style questions

3.1 For $A = \begin{bmatrix} 2 & 1 \\ 0 & 3 \end{bmatrix}$, find an eigenvector for each eigenvalue and check whether $\begin{pmatrix} 1 \\ 1 \end{pmatrix}$ is an eigenvector. [4]

Solutions

2.1 The determinant is $2 \times 3 - 1 \times 0 = 6$.

2.2 The first transformed coordinate is $2 \times 1 + 1 \times 2 = 4$.

2.3 This triangular matrix has eigenvalues given by its diagonal entries 2 and 3, so the larger is 3.

2.4 False. Matrix multiplication is usually not commutative. A determinant of zero means the transformation loses a dimension and has no inverse. Eigenvalues and diagonalisation are advanced scope, not ordinary GCSE vector content.

2.5 A A scalar that determines whether a square matrix is invertible and, in two dimensions, its signed area factor.

3.1 λ_2 : eigenvector $(1, 0)$; λ_3 : $(1, 1)$. $A(1, 1) = (3, 3) = 3(1, 1)$. Nonzero scalar multiples also work; the zero vector is excluded.