

## 5.1 Derivatives and stationary points

Name: \_\_\_\_\_ Class: \_\_\_\_\_ Date: \_\_\_\_\_

Total: 13 marks

**KEY WORDS** derivative

### Objective

Derivatives and stationary points. Supported SL concept preparation; original classroom practice, not an official IB paper or assessed learner work.

### 1 Key knowledge and worked examples

#### ■ What is the slope at one point?

- A curved road has different slopes at different positions. An average gradient cannot describe every point.
- This lesson studies **derivative** : The instantaneous rate of change, also the gradient of a tangent.

#### ■ Choose the mathematical structure

- For  $y = ax^n$ ,  $dy/dx = anx^{n-1}$ . A stationary point satisfies  $dy/dx = 0$ . Check the sign change of the derivative, or the second derivative when it is nonzero, to classify it.
- State the allowed inputs and units before calculating. An equation should express the relationship, not just record a calculator entry.

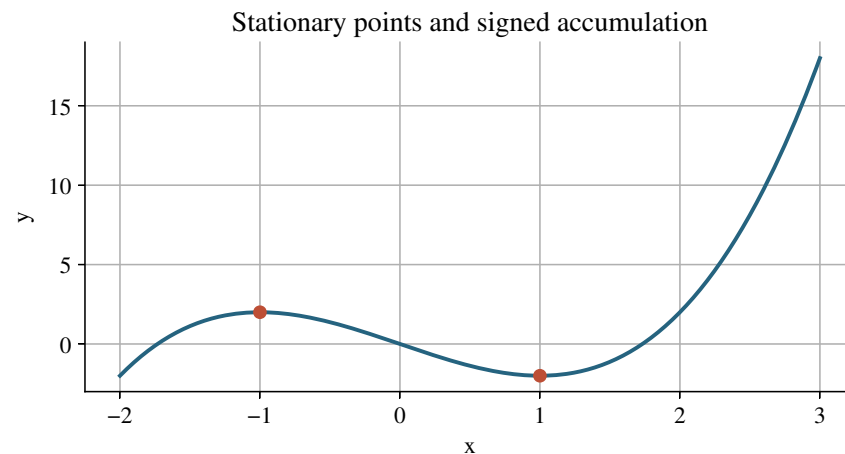
#### ■ Work through a checked case

$$\frac{d}{dx}(ax^n) = anx^{n-1}, \quad f'(x) = 0$$

- Check the result against the starting quantities. Substitute into the original relation, or compare the graph and numerical answer where appropriate.

#### Example:

For  $y = x^3 - 3x$ ,  $dy/dx = 3x^2 - 3$ . At  $x = 1$ , the gradient is 0 and  $y = -2$ . The second derivative is  $6x$ , positive at  $x = 1$ , so this is a local minimum. At  $x = -1$ ,  $y = 2$  and the second derivative is negative, giving a local maximum.



#### ■ Test a tempting shortcut

- A zero derivative does not always mean a maximum or minimum:  $y = x^3$  is stationary at 0 but continues increasing. An endpoint can also produce an extreme value on a restricted domain.
- When a shortcut fails, identify the assumption it breaks. Keep an exact value until the requested final rounding.

#### Warn:

Every point with zero derivative is a local maximum or minimum. This claim is false. Explain which definition or assumption it violates.

#### ■ Interpret a new situation

- At GCSE/IGCSE use only the polynomial scope allowed by the tier; do not add chain, product or quotient rules there. At advanced level, connect the derivative to rates and optimization with a valid domain.
- A complete solution gives the mathematical result and explains what it means. Check that it is possible in the stated context.

■ Use this in your course

- Current first-assessment-2021 Analysis and Approaches SL. This is authored concept support; the full guide is needed to certify every objective.
- Give the method before the final answer, and use the paper's calculator and formula rules. Review a wrong answer by locating the first invalid step.

**Key:**

The instantaneous rate of change, also the gradient of a tangent. Choose the relationship, show the method, check its assumptions and interpret the result.

## 2 Practice

**2.1** For  $y=x^3-3x$ , find  $dy/dx$  at  $x=2$ . Show the relation, substitution and units. [2]

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**2.2** Find the y-coordinate of the stationary point at  $x=1$ . Show the relation, substitution and units. [2]

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**2.3** For  $y=4x^3$ , find  $dy/dx$  at  $x=2$ . Show the relation, substitution and units. [2]

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**2.4** Decide whether this claim is true and justify: Every point with zero derivative is a local maximum or minimum. [2]

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**2.5** Which description correctly defines derivative? Choose one answer.

- **A** The instantaneous rate of change, also the gradient of a tangent.
- **B** A numerical answer that needs no assumptions or interpretation
- **C** A rule that remains valid even when its required conditions fail

[1]

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## 3 Exam-style questions

**3.1** For  $y=x^3-3x$  on  $[-2, 2]$ , find the absolute maximum and minimum, including all candidates. [4]

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