

Calculus · supported focus

International Baccalaureate · IB Diploma · Mathematics: Analysis and Approaches · HL

Scope and prerequisites

Supported HL focus. First assessment 2021; current through 2028. First-assessment-2029 course is separate.. Remaining guide, assessment and practical requirements retain their recorded holds.

Prerequisites: read the stated quantities and units, use arithmetic and the model conditions below. Each lesson develops its own method before independent transfer.

These are original or explicitly fictional teaching examples, not actual measurements or completed assessed learner investigations.

Derivatives and stationary points

■ What is the slope at one point?

- A curved road has different slopes at different positions. An average gradient cannot describe every point.
- This lesson studies **derivative** : The instantaneous rate of change, also the gradient of a tangent.

■ Choose the mathematical structure

- For $y=ax^n$, $dy/dx=nax^{n-1}$. A stationary point satisfies $dy/dx=0$. Check the sign change of the derivative, or the second derivative when it is nonzero, to classify it.
- State the allowed inputs and units before calculating. An equation should express the relationship, not just record a calculator entry.

■ Work through a checked case

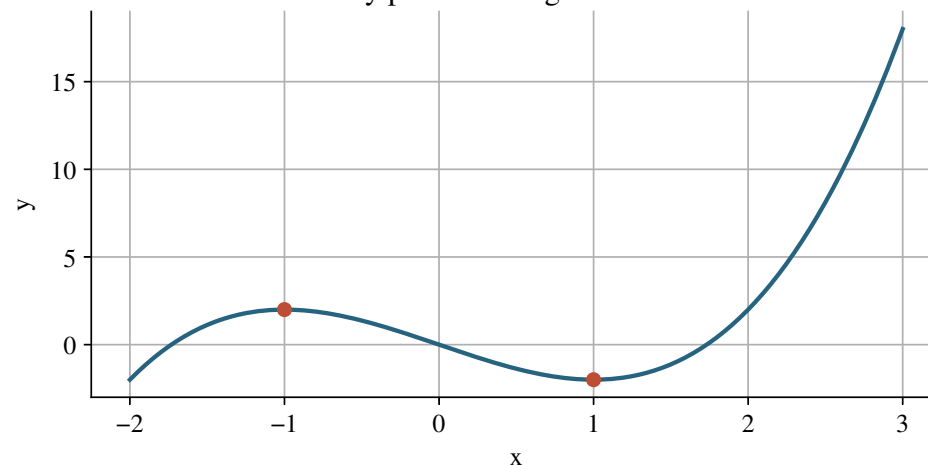
$$\frac{d}{dx}(ax^n) = nax^{n-1}, \quad f'(x) = 0$$

- Check the result against the starting quantities. Substitute into the original relation, or compare the graph and numerical answer where appropriate.

Example:

For $y=x^3-3x$, $dy/dx=3x^2-3$. At $x=1$, the gradient is 0 and $y=-2$. The second derivative is $6x$, positive at $x=1$, so this is a local minimum. At $x=-1$, $y=2$ and the second derivative is negative, giving a local maximum.

Stationary points and signed accumulation



■ Test a tempting shortcut

- A zero derivative does not always mean a maximum or minimum: $y=x^3$ is stationary at 0 but continues increasing. An endpoint can also produce an extreme value on a restricted domain.
- When a shortcut fails, identify the assumption it breaks. Keep an exact value until the requested final rounding.

Warn:

Every point with zero derivative is a local maximum or minimum. This claim is false. Explain which definition or assumption it violates.

■ Interpret a new situation

- At GCSE/IGCSE use only the polynomial scope allowed by the tier; do not add chain, product or quotient rules there. At advanced level, connect the derivative to rates and optimization with a valid domain.
- A complete solution gives the mathematical result and explains what it means. Check that it is possible in the stated context.

■ Use this in your course

- Current first-assessment-2021 Analysis and Approaches HL. This is authored concept support; the full guide is needed to certify every objective.
- Give the method before the final answer, and use the paper's calculator and formula rules. Review a wrong answer by locating the first invalid step.

Key:

The instantaneous rate of change, also the gradient of a tangent. Choose the relationship, show the method, check its assumptions and interpret the result.

Integrals, area and accumulation

■ How much change has accumulated?

- A velocity graph tells us motion at an instant. How can we recover the displacement accumulated over time?
- This lesson studies **antiderivative** : A function whose derivative equals the given integrand.

■ Choose the mathematical structure

- For $n \neq -1$, the integral of ax^n is $ax^{n+1}/(n+1) + C$. A definite integral is signed accumulation. Split at sign changes when total area or distance is required.
- State the allowed inputs and units before calculating. An equation should express the relationship, not just record a calculator entry.

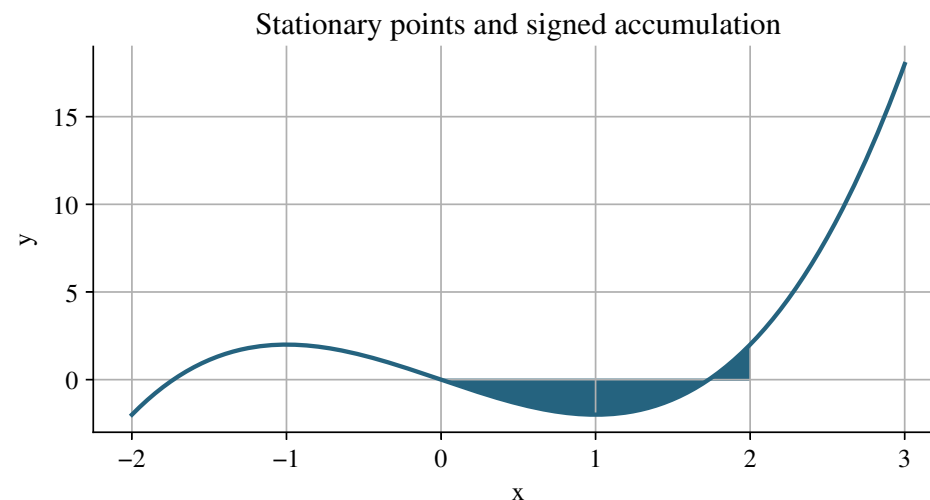
■ Work through a checked case

$$\int ax^n dx = \frac{ax^{n+1}}{n+1} + C \quad (n \neq -1)$$

- Check the result against the starting quantities. Substitute into the original relation, or compare the graph and numerical answer where appropriate.

Example:

For $v(t) = 3t^2 - 3$ over $0 \leq t \leq 2$, displacement = $[t^3 - 3t]_0^2 = 2$. Since v changes sign at $t = 1$, distance = $-[t^3 - 3t]_0^1 + [t^3 - 3t]_1^2 = 2 + 4 = 6$. The constants cancel only for a definite integral.



■ Test a tempting shortcut

- The integral of $1/x$ is $\ln|x| + C$, not the power rule with $n = -1$. Signed area can be zero even when the total enclosed area is positive.
- When a shortcut fails, identify the assumption it breaks. Keep an exact value until the requested final rounding.

Warn:

A definite integral always equals the total positive area, even when the graph crosses the axis. This claim is false. Explain which definition or assumption it violates.

■ Interpret a new situation

- Identify what the integral means and include the correct units. A rate measured per second integrates to the underlying quantity, not to another rate. Differentiate an antiderivative to check it.
- A complete solution gives the mathematical result and explains what it means. Check that it is possible in the stated context.

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Key:
 A function whose derivative equals the given integrand. Choose the relationship, show the method, check its assumptions and interpret the result.

Chain, product, quotient and implicit differentiation

- Does the inside expression change too?
 - A cost curve is a power of a changing expression. Differentiating the outer power alone misses the rate of its input.
 - This lesson studies **chain rule** : The rule that multiplies the outer derivative by the inner derivative for a composite function.

- Choose the mathematical structure
 - For $y=f(g(x))$, $y' = f'(g(x))g'(x)$. For uv , differentiate to $u'v + uv'$. For u/v , use $(u'v - uv')/v^2$, where $v \neq 0$.
 - State the allowed inputs and units before calculating. An equation should express the relationship, not just record a calculator entry.

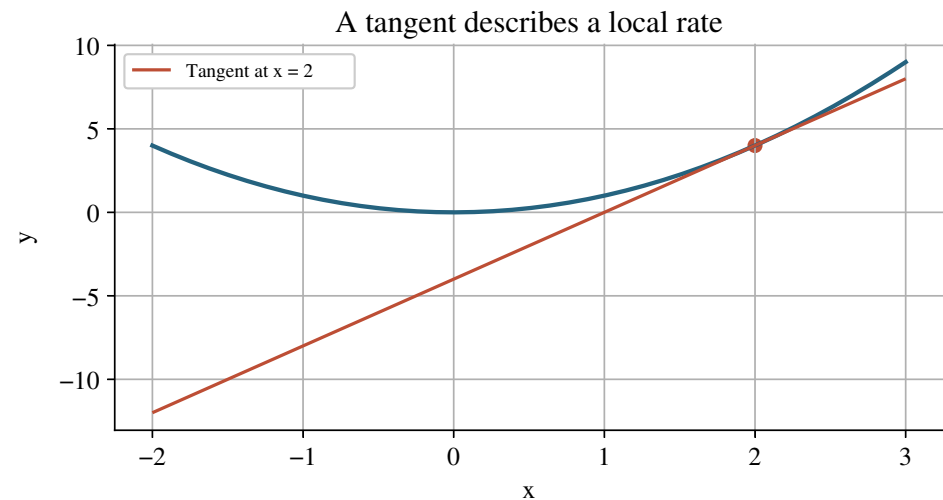
- Work through a checked case

$$\frac{d}{dx}f(g(x)) = f'(g(x))g'(x)$$

- Check the result against the starting quantities. Substitute into the original relation, or compare the graph and numerical answer where appropriate.

Example:

For $y=(3x+1)^4$, $y' = 4(3x+1)^3 \times 3 = 12(3x+1)^3$. At $x=0$, the gradient is 12.
 For $x^2+y^2=25$, differentiate implicitly: $2x+2y y' = 0$, so $y' = -x/y$ when $y \neq 0$.



- Test a tempting shortcut
 - A derivative of a product is not the product of the derivatives. In implicit differentiation, every differentiated function of y brings a dy/dx factor. A quotient's denominator is squared.
 - When a shortcut fails, identify the assumption it breaks. Keep an exact value until the requested final rounding.

Warn:

The derivative of $u(x)v(x)$ is always u' times v' . This claim is false. Explain which definition or assumption it violates.

- Interpret a new situation
 - Choose a useful form before differentiating: expanding a short polynomial may be simpler. For related rates, write the relation in symbols, differentiate with respect to time, then substitute measured values.
 - A complete solution gives the mathematical result and explains what it means. Check that it is possible in the stated context.

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- Give the method before the final answer, and use the paper's calculator and formula rules. Review a wrong answer by locating the first invalid step.

Key:

The rule that multiplies the outer derivative by the inner derivative for a composite function. Choose the relationship, show the method, check its assumptions and interpret the result.

Substitution, parts and partial fractions

■ Can we integrate the two factors separately?

- A simple-looking product such as $x e^x$ cannot be integrated by separately integrating each factor.
- This lesson studies **integration by parts** : An integration method based on the derivative of a product.

■ Choose the mathematical structure

- Use substitution when an inner derivative appears as a factor. By parts, integral $u v' = uv - \int u' v$. For a rational function, divide first if needed, then decompose into partial fractions.
- State the allowed inputs and units before calculating. An equation should express the relationship, not just record a calculator entry.

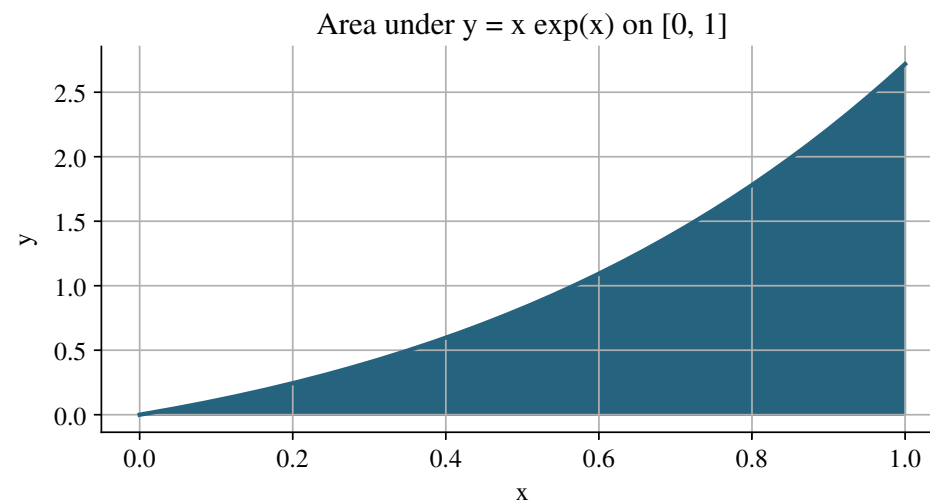
■ Work through a checked case

$$\int u v' dx = uv - \int u' v dx$$

- Check the result against the starting quantities. Substitute into the original relation, or compare the graph and numerical answer where appropriate.

Example:

For integral $x e^x dx$, take $u=x$ and $v' = e^x$. Then the integral is $x e^x - e^x + C$. For integral $2x/(x^2+1) dx$, substitute $w=x^2+1$, $dw=2x dx$; the answer is $\ln(x^2+1)+C$.



■ Test a tempting shortcut

- Choosing u and v' well matters: the remaining integral should become simpler. In a definite substitution, either change the limits or return to x before applying the original limits.
- When a shortcut fails, identify the assumption it breaks. Keep an exact value until the requested final rounding.

Warn:

The integral of a product equals the product of its separate integrals. This claim is false. Explain which definition or assumption it violates.

■ Interpret a new situation

- Check by differentiating. For volumes of revolution around the x -axis use $V = \pi \int y^2 dx$; do not confuse the square of a function with the integral of the function.
- A complete solution gives the mathematical result and explains what it means. Check that it is possible in the stated context.

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Key:

An integration method based on the derivative of a product. Choose the relationship, show the method, check its assumptions and interpret the result.

Differential equations and numerical solutions

■ What does the starting value decide?

- A growing population's rate is proportional to its current size. The rate equation describes a whole family until an initial population is supplied.
- This lesson studies **initial condition** : A specified value that selects a particular solution from a family.

■ Choose the mathematical structure

- For $dy/dx=ky$, separate variables: $dy/y=k dx$, giving $y=Ae^{(kx)}$. Use an initial condition to find A. Euler's method takes $y_{next}=y+h f(x,y)$, with a chosen step h.
- State the allowed inputs and units before calculating. An equation should express the relationship, not just record a calculator entry.

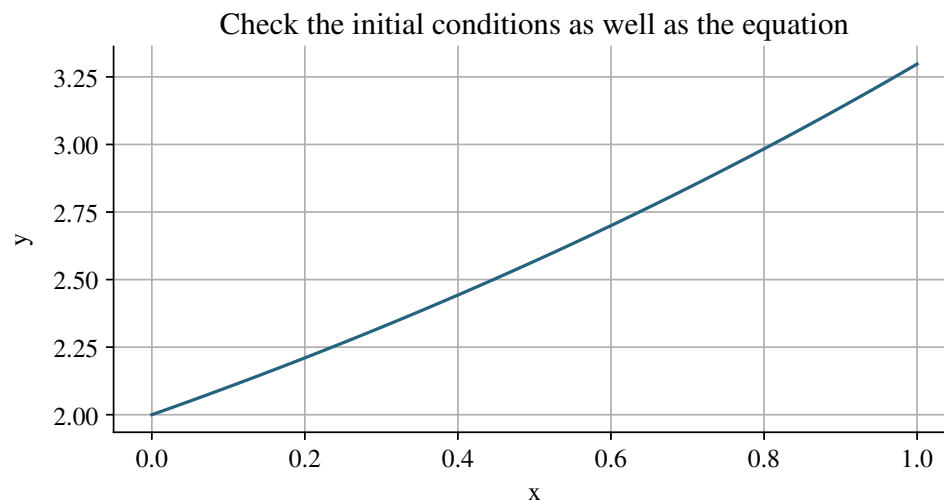
■ Work through a checked case

$$\frac{dy}{dx} = ky, \quad y = Ae^{kx}$$

- Check the result against the starting quantities. Substitute into the original relation, or compare the graph and numerical answer where appropriate.

Example:

With $dy/dx=0.5y$ and $y(0)=2$, $y=2e^{(0.5x)}$. Euler with $h=0.2$ gives $y(0.2)\approx 2+0.2\times 1=2.2$ and $y(0.4)\approx 2.2+0.2\times 1.1=2.42$. The exact second value is about 2.4428.



■ Test a tempting shortcut

- An initial condition determines the integration constant; it is not a replacement for integrating. Dividing by y can omit an equilibrium solution $y=0$. Euler accuracy depends on step size and the equation.
- When a shortcut fails, identify the assumption it breaks. Keep an exact value until the requested final rounding.

Warn:

Every step size gives exactly the same Euler approximation. This claim is false. Explain which definition or assumption it violates.

■ Interpret a new situation

- For second-order linear equations, combine the complementary function with an appropriate particular integral, then apply the required initial conditions. This is Further Pure scope; check the named unit.
- A complete solution gives the mathematical result and explains what it means. Check that it is possible in the stated context.

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- Give the method before the final answer, and use the paper's calculator and formula rules. Review a wrong answer by locating the first invalid step.

Key:

A specified value that selects a particular solution from a family. Choose the relationship, show the method, check its assumptions and interpret the result.