

Geometry and trigonometry · supported focus

International Baccalaureate · IB Diploma · Mathematics: Analysis and Approaches · HL

Scope and prerequisites

Supported HL focus. First assessment 2021; current through 2028. First-assessment-2029 course is separate.. Remaining guide, assessment and practical requirements retain their recorded holds.

Prerequisites: read the stated quantities and units, use arithmetic and the model conditions below. Each lesson develops its own method before independent transfer. These are original or explicitly fictional teaching examples, not actual measurements or completed assessed learner investigations.

Angle reasoning, similarity and mensuration

■ Does volume scale like length?

- A model has lengths one third of the real object. How much smaller are its area and volume?
- This lesson studies **scale factor** : The multiplier that relates corresponding lengths in similar shapes.

■ Choose the mathematical structure

- For similar shapes with length scale factor k , areas scale by k^2 and volumes by k^3 . State angle reasons explicitly. A circle’s tangent is perpendicular to the radius at the contact point.
- State the allowed inputs and units before calculating. An equation should express the relationship, not just record a calculator entry.

■ Work through a checked case

$$\frac{A_2}{A_1} = k^2, \quad \frac{V_2}{V_1} = k^3$$

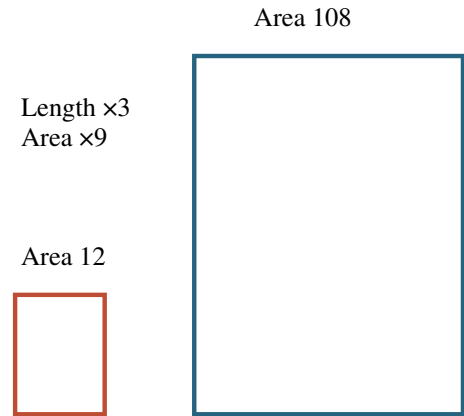
- Check the result against the starting quantities. Substitute into the original relation, or compare the graph and numerical answer where appropriate.

Example:

If model-to-real length factor is 3, a model area of 12 cm^2 gives $12 \times 3^2 = 108\text{ cm}^2$ and a model volume of 8 cm^3 gives $8 \times 3^3 = 216\text{ cm}^3$. A cylinder with $r=3, h=5$ has volume

$$\pi r^2 h = 45\pi.$$

A length scale factor is squared for area



■ Test a tempting shortcut

- Equal angles alone establish similarity, not equal size. Use corresponding lengths in the same order. Convert linear units before calculating area or volume, or square/cube the conversion factor correctly.
- When a shortcut fails, identify the assumption it breaks. Keep an exact value until the requested final rounding.

Warn:

Doubling every length of a solid doubles its volume. This claim is false. Explain which definition or assumption it violates.

■ Interpret a new situation

- A geometric proof should name the relevant theorem, identify the equal angle or ratio, and draw the conclusion. A scale drawing is evidence only when the task permits measurement.
- A complete solution gives the mathematical result and explains what it means. Check that it is possible in the stated context.

■ Use this in your course

- Current first-assessment-2021 Analysis and Approaches HL. This is authored concept support; the full guide is needed to certify every objective.
- Give the method before the final answer, and use the paper's calculator and formula rules. Review a wrong answer by locating the first invalid step.

Key:

The multiplier that relates corresponding lengths in similar shapes. Choose the relationship, show the method, check its assumptions and interpret the result.

Right triangles and non-right triangles

■ Which side does the ladder need?

- A ladder reaches a height of 4 m while its foot is 3 m from a wall. Which sides are known, and which angle do we need?
- This lesson studies **hypotenuse** : The side opposite the right angle in a right-angled triangle.

■ Choose the mathematical structure

- In a right triangle $a^2 + b^2 = c^2$; $\sin\theta = \text{opposite/hypotenuse}$, $\cos\theta = \text{adjacent/hypotenuse}$ and $\tan\theta = \text{opposite/adjacent}$. For other triangles, use the sine or cosine rule, or $\text{area} = ab \sin C/2$.
- State the allowed inputs and units before calculating. An equation should express the relationship, not just record a calculator entry.

■ Work through a checked case

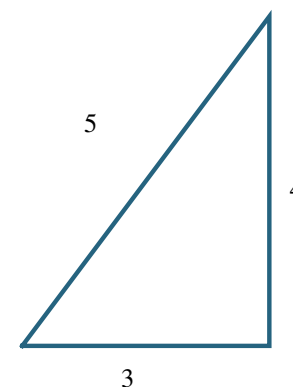
$$a^2 + b^2 = c^2, \quad \tan\theta = \frac{\text{opposite}}{\text{adjacent}}$$

- Check the result against the starting quantities. Substitute into the original relation, or compare the graph and numerical answer where appropriate.

Example:

The ladder length is $c = \sqrt{3^2 + 4^2} = 5$ m. Its angle to the ground satisfies $\tan\theta = 4/3$, so $\theta \approx 53.1^\circ$. With two sides 6 and 8 enclosing 60° , $c^2 = 6^2 + 8^2 - 2 \times 6 \times 8 \cos 60^\circ = 52$.

Label the known sides before choosing a rule



■ Test a tempting shortcut

- Label sides relative to the chosen angle. Pythagoras needs a right angle. A calculator angle mode error can produce a plausible but wrong result. Keep unrounded values for later steps.
- When a shortcut fails, identify the assumption it breaks. Keep an exact value until the requested final rounding.

Warn:

Pythagoras applies to every triangle, including triangles without a right angle. This claim is false. Explain which definition or assumption it violates.

■ Interpret a new situation

- Use a plan or elevation for a three-dimensional problem before applying a triangle rule. Explain why the chosen triangle contains the required length or angle.
- A complete solution gives the mathematical result and explains what it means. Check that it is possible in the stated context.

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- Give the method before the final answer, and use the paper's calculator and formula rules. Review a wrong answer by locating the first invalid step.

Key:

The side opposite the right angle in a right-angled triangle. Choose the relationship, show the method, check its assumptions and interpret the result.

Radians, identities and trigonometric equations

■ How far does the rim travel?

- A rotating wheel sweeps an arc. Radians let its angle connect directly to the distance travelled along the rim.
- This lesson studies **radian** : The angle subtended by an arc whose length equals the radius.

■ Choose the mathematical structure

- For θ in radians, arc length $s=r\theta$ and sector area $A=r^2\theta/2$. Use $\sin^2\theta+\cos^2\theta=1$ and angle identities to simplify or solve. List every solution in the stated interval.
- State the allowed inputs and units before calculating. An equation should express the relationship, not just record a calculator entry.

■ Work through a checked case

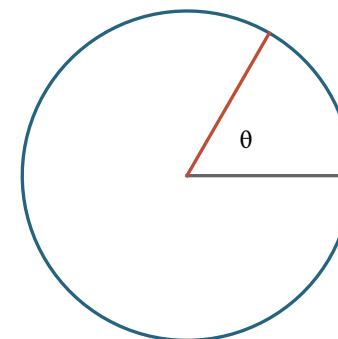
$$s = r\theta, \quad A = \frac{1}{2}r^2\theta, \quad \sin^2\theta + \cos^2\theta = 1$$

- Check the result against the starting quantities. Substitute into the original relation, or compare the graph and numerical answer where appropriate.

Example:

For $r=4$ and $\theta=\pi/3$, $s=4\pi/3$ and $A=8\pi/3$. Solving $\sin x=1/2$ on $0\leq x<2\pi$ gives $x=\pi/6$ and $5\pi/6$. For $\sin 2x=1/2$, solve first on $0\leq 2x<4\pi$ before dividing by 2.

A radius fixes distance from the centre



radius $\times$ angle gives arc length

■ Test a tempting shortcut

- A calculator's inverse sine gives one principal value. It does not give every solution. Dividing by $\sin x$ can discard solutions where $\sin x=0$; check that case first.
- When a shortcut fails, identify the assumption it breaks. Keep an exact value until the requested final rounding.

Warn:

The principal inverse sine value is always the only solution in a full turn. This claim is false. Explain which definition or assumption it violates.

■ Interpret a new situation

- Choose an identity that reduces the equation to one trigonometric function. Keep exact familiar angles where possible, and check each solution against the original equation.
- A complete solution gives the mathematical result and explains what it means. Check that it is possible in the stated context.

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- Give the method before the final answer, and use the paper's calculator and formula rules. Review a wrong answer by locating the first invalid step.

Key:
The angle subtended by an arc whose length equals the radius. Choose the relationship, show the method, check its assumptions and interpret the result.

Vectors and transformation geometry

- **Why is displacement shorter than the walk?**
 - Walking 4 m east and 3 m north gives a displacement of 5 m, even though the travelled distance is 7 m.
 - This lesson studies **resultant** : The vector sum representing the combined displacement or force.

- **Choose the mathematical structure**
 - Add corresponding vector components. A displacement from A to B is OB-OA. Parallel vectors are scalar multiples. A vector has magnitude and direction; a distance is a scalar.
 - State the allowed inputs and units before calculating. An equation should express the relationship, not just record a calculator entry.

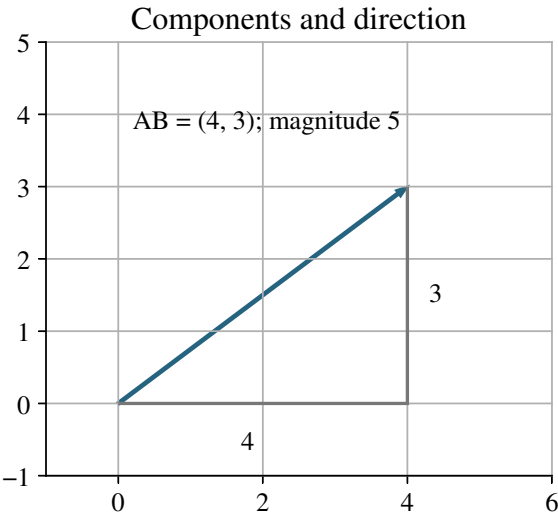
- **Work through a checked case**

$$\overrightarrow{AB} = \mathbf{b} - \mathbf{a}, \quad |\mathbf{v}| = \sqrt{v_x^2 + v_y^2}$$

- Check the result against the starting quantities. Substitute into the original relation, or compare the graph and numerical answer where appropriate.

Example:

With $\mathbf{a}=(4,1)$ and $\mathbf{b}=(1,3)$, $\mathbf{a}+\mathbf{b}=(5,4)$. Their scalar product is $\mathbf{a} \cdot \mathbf{b}=4\times 1+1\times 3=7$. The magnitude of $\mathbf{a}$ is $\sqrt{17}$. For $\mathbf{A}=(1,2)$, $\mathbf{B}=(5,5)$, $\mathbf{AB}=(4,3)$ and $|\mathbf{AB}|=5$.



- **Test a tempting shortcut**
 - The order of subtraction matters: $\mathbf{BA}=-\mathbf{AB}$. Proving parallelism needs a scalar-multiple relation; a sketch alone is insufficient. Negative enlargement reverses position about its centre.
 - When a shortcut fails, identify the assumption it breaks. Keep an exact value until the requested final rounding.

Warn:

$\mathbf{AB}$ and $\mathbf{BA}$ always have the same components. This claim is false. Explain which definition or assumption it violates.

- **Interpret a new situation**
 - For a translation, every point uses the same displacement. For advanced line problems, distinguish the position vector from a direction vector and solve parameter equations consistently.
 - A complete solution gives the mathematical result and explains what it means. Check that it is possible in the stated context.

■ **Use this in your course**

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- Give the method before the final answer, and use the paper's calculator and formula rules. Review a wrong answer by locating the first invalid step.

Key:

The vector sum representing the combined displacement or force. Choose the relationship, show the method, check its assumptions and interpret the result.