

5.2 Integrals, area and accumulation

Name: _____ Class: _____ Date: _____ Total: 13 marks

KEY WORDS antiderivative

Objective

Integrals, area and accumulation. Supported HL concept preparation; original classroom practice, not an official IB paper or assessed learner work.

1 Key knowledge and worked examples

■ How much change has accumulated?

- A velocity graph tells us motion at an instant. How can we recover the displacement accumulated over time?
- This lesson studies **antiderivative** : A function whose derivative equals the given integrand.

■ Choose the mathematical structure

- For $n \neq -1$, the integral of ax^n is $ax^{n+1}/(n+1) + C$. A definite integral is signed accumulation. Split at sign changes when total area or distance is required.
- State the allowed inputs and units before calculating. An equation should express the relationship, not just record a calculator entry.

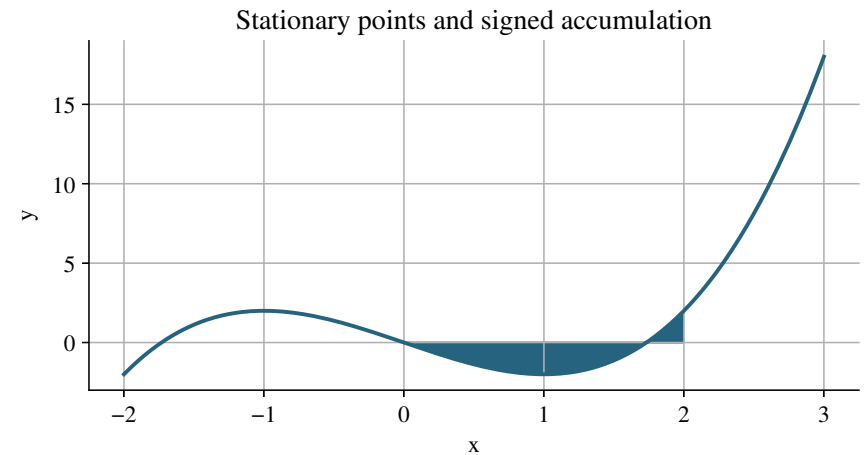
■ Work through a checked case

$$\int ax^n dx = \frac{ax^{n+1}}{n+1} + C \quad (n \neq -1)$$

- Check the result against the starting quantities. Substitute into the original relation, or compare the graph and numerical answer where appropriate.

Example:

For $v(t) = 3t^2 - 3$ over $0 \leq t \leq 2$, displacement = $[t^3 - 3t]_0^2 = 2$. Since v changes sign at $t = 1$, distance = $-[t^3 - 3t]_0^1 + [t^3 - 3t]_1^2 = 2 + 4 = 6$. The constants cancel only for a definite integral.



■ Test a tempting shortcut

- The integral of $1/x$ is $\ln|x| + C$, not the power rule with $n = -1$. Signed area can be zero even when the total enclosed area is positive.
- When a shortcut fails, identify the assumption it breaks. Keep an exact value until the requested final rounding.

Warn:

A definite integral always equals the total positive area, even when the graph crosses the axis. This claim is false. Explain which definition or assumption it violates.

■ Interpret a new situation

- Identify what the integral means and include the correct units. A rate measured per second integrates to the underlying quantity, not to another rate. Differentiate an antiderivative to check it.
- A complete solution gives the mathematical result and explains what it means. Check that it is possible in the stated context.

■ Use this in your course

- Current first-assessment-2021 Analysis and Approaches HL. This is authored concept support; the full guide is needed to certify every objective.
- Give the method before the final answer, and use the paper's calculator and formula rules. Review a wrong answer by locating the first invalid step.

Key:

A function whose derivative equals the given integrand. Choose the relationship, show the method, check its assumptions and interpret the result.

2 Practice

2.1 Find the integral of $3x^2$ from 0 to 2. Show the relation, substitution and units. [2]

2.2 For $v=3t^2-3$, find displacement from $t=0$ to 2. Show the relation, substitution and units. [2]

2.3 For that motion, find total distance from $t=0$ to 2. Show the relation, substitution and units. [2]

2.4 Decide whether this claim is true and justify: A definite integral always equals the total positive area, even when the graph crosses the axis. [2]

2.5 Which description correctly defines antiderivative? Choose one answer.

- **A** A function whose derivative equals the given integrand.
 - **B** A numerical answer that needs no assumptions or interpretation
 - **C** A rule that remains valid even when its required conditions fail
- [1]

3 Exam-style questions

3.1 For velocity $v=t-2$ on $0 \leq t \leq 5$, find displacement and total distance and explain the distinction. [4]

Solutions

2.1 An antiderivative is x^3 . Subtract its values: $2^3 - 0^3 = 8$.

2.2 Displacement = $[t^3 - 3t]$ from 0 to 2 = $8 - 6 = 2$.

2.3 Velocity changes sign at $t = 1$. The two positive distances are 2 and 4, giving 6.

2.4 False. The integral of $1/x$ is $\ln|x| + C$, not the power rule with $n = -1$. Signed area can be zero even when the total enclosed area is positive.

2.5 A A function whose derivative equals the given integrand.

3.1 $\text{Integral}[t^2/2 - 2t]_0^5 = 2.5$. Sign changes at 2; signed integrals are -2 then 4.5, so distance 6.5. Units follow velocity/time units. Distance integrates $|v|$.