

4.2 Probability, trees and conditional reasoning

Name: _____ Class: _____ Date: _____

Total: 13 marks

KEY WORDS conditional probability

Objective

Probability, trees and conditional reasoning. Supported HL concept preparation; original classroom practice, not an official IB paper or assessed learner work.

1 Key knowledge and worked examples

■ What changes after the first draw?

- A bag has 3 red and 2 blue counters. Taking two without replacement changes the chance of the second colour.
- This lesson studies **conditional probability** : The probability of an event after restricting the sample space to a stated condition.

■ Choose the mathematical structure

- Multiply along a tree branch and add disjoint branches. With replacement, the composition stays fixed. Conditional probability is $P(A \text{ given } B) = P(A \cap B) / P(B)$, for $P(B) > 0$.
- State the allowed inputs and units before calculating. An equation should express the relationship, not just record a calculator entry.

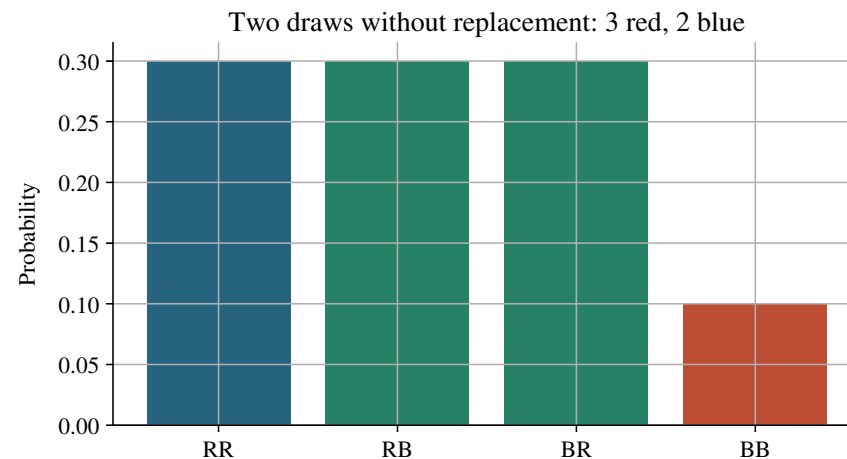
■ Work through a checked case

$$P(A | B) = \frac{P(A \cap B)}{P(B)}, \quad P(B) > 0$$

- Check the result against the starting quantities. Substitute into the original relation, or compare the graph and numerical answer where appropriate.

Example:

Without replacement, $P(\text{two red}) = 3/5 \times 2/4 = 3/10$. $P(\text{one of each}) = 3/5 \times 2/4 + 2/5 \times 3/4 = 3/5$. If $P(A \cap B) = 0.12$ and $P(B) = 0.3$, $P(A \text{ given } B) = 0.4$.



■ Test a tempting shortcut

- Mutually exclusive means no overlap; independent means that knowing one event does not change the other's probability. Two disjoint events with positive probability are not independent.
- When a shortcut fails, identify the assumption it breaks. Keep an exact value until the requested final rounding.

Warn:

Mutually exclusive events with positive probabilities must be independent. This claim is false. Explain which definition or assumption it violates.

■ Interpret a new situation

- A two-way table makes the restricted denominator visible. Before using a product $P(A)P(B)$, justify independence from the context or the supplied information.
- A complete solution gives the mathematical result and explains what it means. Check that it is possible in the stated context.

■ Use this in your course

- Current first-assessment-2021 Analysis and Approaches HL. This is authored concept support; the full guide is needed to certify every objective.
- Give the method before the final answer, and use the paper's calculator and formula rules. Review a wrong answer by locating the first invalid step.

Key:

The probability of an event after restricting the sample space to a stated condition. Choose the relationship, show the method, check its assumptions and interpret the result.

2 Practice

2.1 Find the probability of two red from 3 red and 2 blue without replacement. Show the relation, substitution and units. [2]

2.2 Find $P(A \text{ given } B)$ if $P(A \cap B)=0.12$ and $P(B)=0.3$. Show the relation, substitution and units. [2]

2.3 Find the probability of one red and one blue in that bag. Show the relation, substitution and units. [2]

2.4 Decide whether this claim is true and justify: Mutually exclusive events with positive probabilities must be independent. [2]

2.5 Which description correctly defines conditional probability? Choose one answer.

- **A** The probability of an event after restricting the sample space to a stated condition.
- **B** A numerical answer that needs no assumptions or interpretation
- **C** A rule that remains valid even when its required conditions fail

[1]

3 Exam-style questions

3.1 A bag has 3 red and 2 blue counters. Draw two without replacement. Given at least one red, find the probability both are red. [4]

Solutions

2.1 Without replacement, $P(RR) = 3/5 \times 2/4 = 0.3$.

2.2 $P(A|B) = P(A \cap B) / P(B) = 0.12 / 0.3 = 0.4$.

2.3 Add the two orders: $3/5 \times 2/4 + 2/5 \times 3/4 = 0.6$.

2.4 False. Mutually exclusive means no overlap; independent means that knowing one event does not change the other's probability. Two disjoint events with positive probability are not independent.

2.5 A The probability of an event after restricting the sample space to a stated condition.

3.1 $P(RR) = 3/5 \times 2/4 = 0.3$. $P(\text{at least one } R) = 1 - 2/5 \times 1/4 = 0.9$. Conditional probability $= 0.3 / 0.9 = 1/3$. The conditioning changes the sample space.