

3.2 Right triangles and non-right triangles

Name: _____ Class: _____ Date: _____

Total: 13 marks

KEY WORDS hypotenuse

Objective

Right triangles and non-right triangles. Supported HL concept preparation; original classroom practice, not an official IB paper or assessed learner work.

1 Key knowledge and worked examples

■ Which side does the ladder need?

- A ladder reaches a height of 4 m while its foot is 3 m from a wall. Which sides are known, and which angle do we need?
- This lesson studies **hypotenuse** : The side opposite the right angle in a right-angled triangle.

■ Choose the mathematical structure

- In a right triangle $a^2 + b^2 = c^2$; $\sin\theta = \text{opposite/hypotenuse}$, $\cos\theta = \text{adjacent/hypotenuse}$ and $\tan\theta = \text{opposite/adjacent}$. For other triangles, use the sine or cosine rule, or $\text{area} = ab \sin C/2$.
- State the allowed inputs and units before calculating. An equation should express the relationship, not just record a calculator entry.

■ Work through a checked case

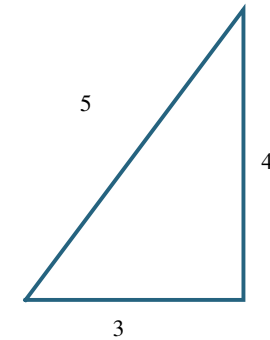
$$a^2 + b^2 = c^2, \quad \tan\theta = \frac{\text{opposite}}{\text{adjacent}}$$

- Check the result against the starting quantities. Substitute into the original relation, or compare the graph and numerical answer where appropriate.

Example:

The ladder length is $c = \sqrt{(3^2 + 4^2)} = 5$ m. Its angle to the ground satisfies $\tan\theta = 4/3$, so $\theta \approx 53.1^\circ$. With two sides 6 and 8 enclosing 60° , $c^2 = 6^2 + 8^2 - 2 \times 6 \times 8 \cos 60^\circ = 52$.

Label the known sides before choosing a rule



■ Test a tempting shortcut

- Label sides relative to the chosen angle. Pythagoras needs a right angle. A calculator angle mode error can produce a plausible but wrong result. Keep unrounded values for later steps.
- When a shortcut fails, identify the assumption it breaks. Keep an exact value until the requested final rounding.

Warn:

Pythagoras applies to every triangle, including triangles without a right angle. This claim is false. Explain which definition or assumption it violates.

■ Interpret a new situation

- Use a plan or elevation for a three-dimensional problem before applying a triangle rule. Explain why the chosen triangle contains the required length or angle.
- A complete solution gives the mathematical result and explains what it means. Check that it is possible in the stated context.

■ Use this in your course

- Current first-assessment-2021 Analysis and Approaches HL. This is authored concept support; the full guide is needed to certify every objective.
- Give the method before the final answer, and use the paper's calculator and formula rules. Review a wrong answer by locating the first invalid step.

Key:

The side opposite the right angle in a right-angled triangle. Choose the relationship, show the method, check its assumptions and interpret the result.

2 Practice

2.1 Find the hypotenuse when the legs are 3 and 4. Show the relation, substitution and units. [2]

2.2 Find $\sin\theta$ when opposite=3 and hypotenuse=5. Show the relation, substitution and units. [2]

2.3 Two sides 6 and 8 enclose 90° . Find their area. Show the relation, substitution and units. [2]

2.4 Decide whether this claim is true and justify: Pythagoras applies to every triangle, including triangles without a right angle. [2]

2.5 Which description correctly defines hypotenuse? Choose one answer.

- **A** The side opposite the right angle in a right-angled triangle.
- **B** A numerical answer that needs no assumptions or interpretation
- **C** A rule that remains valid even when its required conditions fail

[1]

3 Exam-style questions

3.1 A triangle has sides $a=7$, $b=10$ and angle $A=30^\circ$. Determine how many triangles are possible and give the possible B angles. [4]

Solutions

2.1 Pythagoras gives $\sqrt{3^2+4^2}=5$.

2.2 Sine=opposite/hypotenuse= $3/5=0.6$.

2.3 The sides are perpendicular, so area= $6\times 8/2=24$.

2.4 False. Label sides relative to the chosen angle. Pythagoras needs a right angle. A calculator angle mode error can produce a plausible but wrong result. Keep unrounded values for later steps.

2.5 A The side opposite the right angle in a right-angled triangle.

3.1 Sine rule gives $\sin B=10 \sin 30^\circ/7=5/7$. $B\approx 45.585^\circ$ or 134.415° ; both leave a positive third angle. The ambiguous SSA case has two solutions, unlike a right-triangle shortcut.