

## 3.1 Angle reasoning, similarity and mensuration

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Name: \_\_\_\_\_ Class: \_\_\_\_\_ Date: \_\_\_\_\_

**Total: 13 marks**

**KEY WORDS** scale factor

### Objective

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Angle reasoning, similarity and mensuration. Supported HL concept preparation; original classroom practice, not an official IB paper or assessed learner work.

### 1 Key knowledge and worked examples

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#### ■ Does volume scale like length?

- A model has lengths one third of the real object. How much smaller are its area and volume?
  - This lesson studies **scale factor** : The multiplier that relates corresponding lengths in similar shapes.
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#### ■ Choose the mathematical structure

- For similar shapes with length scale factor  $k$ , areas scale by  $k^2$  and volumes by  $k^3$ . State angle reasons explicitly. A circle's tangent is perpendicular to the radius at the contact point.
  - State the allowed inputs and units before calculating. An equation should express the relationship, not just record a calculator entry.
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#### ■ Work through a checked case

$$\frac{A_2}{A_1} = k^2, \quad \frac{V_2}{V_1} = k^3$$

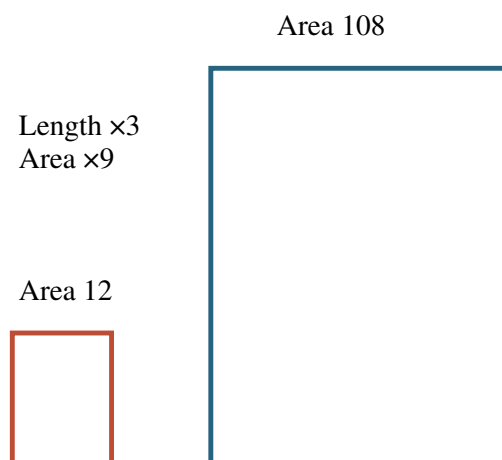
- Check the result against the starting quantities. Substitute into the original relation, or compare the graph and numerical answer where appropriate.

#### Example:

If model-to-real length factor is 3, a model area of  $12 \text{ cm}^2$  gives  $12 \times 3^2 = 108 \text{ cm}^2$  and a model volume of  $8 \text{ cm}^3$  gives  $8 \times 3^3 = 216 \text{ cm}^3$ . A cylinder with  $r=3, h=5$  has volume

$$\pi r^2 h = 45\pi.$$

A length scale factor is squared for area



### ■ Test a tempting shortcut

- Equal angles alone establish similarity, not equal size. Use corresponding lengths in the same order. Convert linear units before calculating area or volume, or square/cube the conversion factor correctly.
- When a shortcut fails, identify the assumption it breaks. Keep an exact value until the requested final rounding.

### Warn:

Doubling every length of a solid doubles its volume. This claim is false. Explain which definition or assumption it violates.

### ■ Interpret a new situation

- A geometric proof should name the relevant theorem, identify the equal angle or ratio, and draw the conclusion. A scale drawing is evidence only when the task permits measurement.
- A complete solution gives the mathematical result and explains what it means. Check that it is possible in the stated context.

### ■ Use this in your course

- Current first-assessment-2021 Analysis and Approaches HL. This is authored concept support; the full guide is needed to certify every objective.
- Give the method before the final answer, and use the paper's calculator and formula rules. Review a wrong answer by locating the first invalid step.

### Key:

The multiplier that relates corresponding lengths in similar shapes. Choose the relationship, show the method, check its assumptions and interpret the result.

## 2 Practice

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**2.1** Two similar figures have length factor 3. Find the area factor. Show the relation, substitution and units. [2]

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**2.2** Two similar solids have length factor 3. Find the volume factor. Show the relation, substitution and units. [2]

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**2.3** Find the sum of interior angles of a pentagon in degrees. Show the relation, substitution and units. [2]

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**2.4** Decide whether this claim is true and justify: Doubling every length of a solid doubles its volume. [2]

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**2.5** Which description correctly defines scale factor? Choose one answer.

- **A** The multiplier that relates corresponding lengths in similar shapes.
- **B** A numerical answer that needs no assumptions or interpretation
- **C** A rule that remains valid even when its required conditions fail [1]

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### 3 Exam-style questions

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**3.1** Two similar closed solids have volume ratio 27:8. Find length and area ratios; explain why doubling a length would not double volume. [4]

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## Solutions

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**2.1** Area scales by the square of the length factor:  $3^2=9$ .

**2.2** Volume scales by the cube:  $3^3=27$ .

**2.3** Interior-angle sum  $= (5-2) \times 180 = 540^\circ$ .

**2.4** False. Equal angles alone establish similarity, not equal size. Use corresponding lengths in the same order. Convert linear units before calculating area or volume, or square/cube the conversion factor correctly.

**2.5 A** The multiplier that relates corresponding lengths in similar shapes.

**3.1** Length ratio 3:2, area 9:4. Area scales as the square and volume as the cube of the length factor, assuming genuine geometric similarity.