

2.2 Exponentials, logarithms and modelling

Name: _____ Class: _____ Date: _____

Total: 13 marks

KEY WORDS half-life

Objective

Exponentials, logarithms and modelling. Supported HL concept preparation; original classroom practice, not an official IB paper or assessed learner work.

1 Key knowledge and worked examples

■ Why does a decay model stay positive?

- A medicine concentration falls by the same percentage each hour. A constant subtraction would eventually predict a negative amount.
- This lesson studies **half-life** : The time for a decaying quantity to fall to half its initial value.

■ Choose the mathematical structure

- For $y = Ae^{kt}$, k is a proportional rate. Taking logs gives $\ln y = \ln A + kt$. Logarithms require positive arguments, and $\log(x+y)$ is not $\log x + \log y$.
- State the allowed inputs and units before calculating. An equation should express the relationship, not just record a calculator entry.

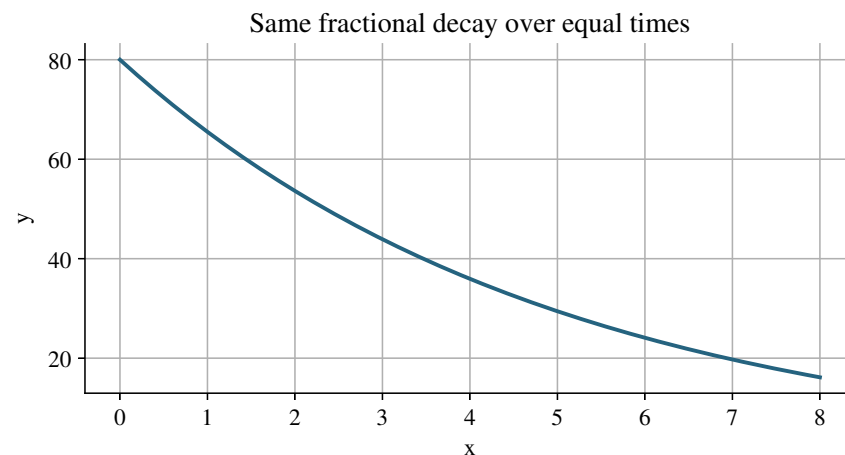
■ Work through a checked case

$$y = Ae^{kt}, \quad \ln y = \ln A + kt$$

- Check the result against the starting quantities. Substitute into the original relation, or compare the graph and numerical answer where appropriate.

Example:

If $y = 80e^{-0.2t}$, $y = 40$ gives $e^{-0.2t} = 1/2$. Hence $t = \ln 2 / 0.2 \approx 3.466$. For $3^x = 20$, $x = \ln 20 / \ln 3$. Plotting $\ln y$ against t linearises this exponential model.



■ Test a tempting shortcut

- A fitted exponential is a model, not a guarantee. Specify the time units and range of use. A negative k describes decay; a negative starting amount usually contradicts the context.
- When a shortcut fails, identify the assumption it breaks. Keep an exact value until the requested final rounding.

Warn:

For positive x and y , $\ln(x+y) = \ln x + \ln y$. This claim is false. Explain which definition or assumption it violates.

■ Interpret a new situation

- Compare actual observations with the model. Systematic departures may indicate changing conditions. In a report, explain what the rate and initial value mean, rather than giving a bare equation.
- A complete solution gives the mathematical result and explains what it means. Check that it is possible in the stated context.

■ Use this in your course

- Current first-assessment-2021 Analysis and Approaches HL. This is authored concept support; the full guide is needed to certify every objective.
- Give the method before the final answer, and use the paper's calculator and formula rules. Review a wrong answer by locating the first invalid step.

Key:

The time for a decaying quantity to fall to half its initial value. Choose the relationship, show the method, check its assumptions and interpret the result.

2 Practice

2.1 Solve $2^x = 32$. Show the relation, substitution and units. [2]

2.2 Find $\ln(e^3)$. Show the relation, substitution and units. [2]

2.3 Find the half-life of $80e^{(-0.2t)}$, to three decimal places. Show the relation, substitution and units. [2]

2.4 Decide whether this claim is true and justify: For positive x and y , $\ln(x+y) = \ln x + \ln y$. [2]

2.5 Which description correctly defines half-life? Choose one answer.

- **A** The time for a decaying quantity to fall to half its initial value.
- **B** A numerical answer that needs no assumptions or interpretation
- **C** A rule that remains valid even when its required conditions fail

[1]

3 Exam-style questions

3.1 A fictional quantity follows $80 \exp(-0.2 t)$. Find when it reaches 20, retaining an exact expression; explain why the half-life does not depend on 80. [4]

Solutions

2.1 $32=2^5$, so $x=5$.

2.2 The natural logarithm undoes exponentiation: $\ln(e^3)=3$.

2.3 Set $e^{(-0.2t)}=1/2$. Then $t=\ln(2)/0.2\approx 3.466$ to three decimal places.

2.4 False. A fitted exponential is a model, not a guarantee. Specify the time units and range of use. A negative k describes decay; a negative starting amount usually contradicts the context.

2.5 A The time for a decaying quantity to fall to half its initial value.

3.1 $t=\ln 4/0.2\approx 6.931$ time units. Half-life $\ln 2/0.2$ follows from a ratio $1/2$; the initial multiplier cancels. Extrapolation assumes the exponential model continues.