

1.6 Binomial expansion and valid approximations

Name: _____ Class: _____ Date: _____

Total: 13 marks

KEY WORDS binomial coefficient

Objective

Binomial expansion and valid approximations. Supported HL concept preparation; original classroom practice, not an official IB paper or assessed learner work.

1 Key knowledge and worked examples

■ How does a small change affect a power?

- A small measurement change affects a power of a quantity. An expansion can show the size of first and second effects.
- This lesson studies **binomial coefficient** : The number of ways to choose a specified number of objects from a set.

■ Choose the mathematical structure

- For positive integer n , $(a+b)^n$ is a finite binomial expansion. For noninteger n , expand $(1+x)^n$ as $1+nx+\frac{n(n-1)}{2}x^2+\dots$ with $|x|<1$. Factor out constants before using this form.
- State the allowed inputs and units before calculating. An equation should express the relationship, not just record a calculator entry.

■ Work through a checked case

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2}x^2 + \dots$$

- Check the result against the starting quantities. Substitute into the original relation, or compare the graph and numerical answer where appropriate.

Example:

$(1+2x)^5=1+10x+40x^2+80x^3+80x^4+32x^5$. For $(1+x)^{-1}$, the first three terms are $1-x+x^2$; at $x=0.1$ this gives 0.91 versus the exact $1/1.1\approx 0.909091$.

$$(1+2x)^5$$

Coefficients: 1, 10, 40, 80, 80, 32

The x^2 coefficient includes $(2x)^2$

■ Test a tempting shortcut

- An expansion in $2x$ requires $|2x|<1$ for the infinite series, not merely $|x|<1$. Positive-integer expansions are finite and do not have that convergence restriction.
- When a shortcut fails, identify the assumption it breaks. Keep an exact value until the requested final rounding.

Warn:

Every binomial expansion is a finite polynomial, including powers that are not positive integers. This claim is false. Explain which definition or assumption it violates.

■ Interpret a new situation

- State the range of validity alongside an approximation. Retain enough terms to support the requested accuracy, and distinguish a coefficient from the whole term containing x .
- A complete solution gives the mathematical result and explains what it means. Check that it is possible in the stated context.

■ Use this in your course

- Current first-assessment-2021 Analysis and Approaches HL. This is authored concept support; the full guide is needed to certify every objective.
- Give the method before the final answer, and use the paper's calculator and formula rules. Review a wrong answer by locating the first invalid step.

Key:

The number of ways to choose a specified number of objects from a set. Choose the relationship, show the method, check its assumptions and interpret the result.

2 Practice

2.1 Find the coefficient of x^2 in $(1+2x)^5$. Show the relation, substitution and units. [2]

2.2 Use $1-x+x^2$ to estimate $1/(1+x)$ at $x=0.1$. Show the relation, substitution and units. [2]

2.3 How many terms are in the full expansion of $(a+b)^5$? Show the relation, substitution and units. [2]

2.4 Decide whether this claim is true and justify: Every binomial expansion is a finite polynomial, including powers that are not positive integers. [2]

2.5 Which description correctly defines binomial coefficient? Choose one answer.

- **A** The number of ways to choose a specified number of objects from a set.
 - **B** A numerical answer that needs no assumptions or interpretation
 - **C** A rule that remains valid even when its required conditions fail
- [1]

3 Exam-style questions

3.1 Use $1-x+x^2$ to approximate $1/(1+x)$ at $x=0.2$. Find the exact error algebraically and explain its sign. [4]

Solutions

2.1 Choose two of five factors and include 2^2 : $C(5,2) \times 4 = 40$.

2.2 Substitute 0.1: $1 - 0.1 + 0.01 = 0.91$.

2.3 The powers run from 0 through 5, giving six terms.

2.4 False. An expansion in $2x$ requires $|2x| < 1$ for the infinite series, not merely $|x| < 1$. Positive-integer expansions are finite and do not have that convergence restriction.

2.5 A The number of ways to choose a specified number of objects from a set.

3.1 Approximation 0.84; exact $5/6$. Identity $(1 - x + x^2) - 1/(1 + x) = x^3/(1 + x)$, so error $1/150 \approx 0.006667$, positive. The infinite geometric expansion requires $|x| < 1$.