

1.5 Complex numbers and roots

Name: _____ Class: _____ Date: _____

Total: 13 marks

KEY WORDS complex conjugate

Objective

Complex numbers and roots. Supported HL concept preparation; original classroom practice, not an official IB paper or assessed learner work.

1 Key knowledge and worked examples

■ What if the root is not real?

- The equation $x^2+1=0$ has no real roots. Extending the number system lets us represent and calculate with both roots.
- This lesson studies **complex conjugate** : The number obtained by reversing the sign of the imaginary part.

■ Choose the mathematical structure

- Write $z=a+bi$ with $i^2=-1$. The conjugate is $a-bi$ and z times its conjugate= a^2+b^2 . The modulus is $\sqrt{a^2+b^2}$; the argument needs the correct quadrant. Conjugate roots occur for polynomials with real coefficients.
- State the allowed inputs and units before calculating. An equation should express the relationship, not just record a calculator entry.

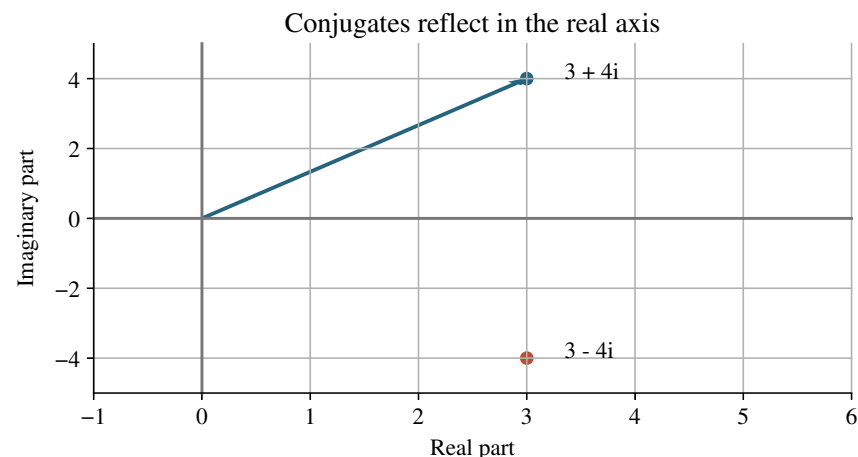
■ Work through a checked case

$$z = a + bi, \quad |z| = \sqrt{a^2 + b^2}, \quad z\bar{z} = a^2 + b^2$$

- Check the result against the starting quantities. Substitute into the original relation, or compare the graph and numerical answer where appropriate.

Example:

For $z=3+4i$, $|z|=5$ and z conjugate= $3-4i$. The reciprocal is $(3-4i)/25$. For $x^2-6x+13=0$, the roots are $3\pm 2i$. Multiplying by the conjugate makes a complex denominator real.



■ Test a tempting shortcut

- The square root of a sum is not the sum of the square roots. An inverse tangent alone may return the wrong quadrant. Do not confuse modulus with squared modulus.
- When a shortcut fails, identify the assumption it breaks. Keep an exact value until the requested final rounding.

Warn:

For $a+bi$, its modulus is always $a+b$. This claim is false. Explain which definition or assumption it violates.

■ Interpret a new situation

- Draw the real axis horizontally and imaginary axis vertically. For products, moduli multiply and arguments add; powers extend this geometric pattern through de Moivre's theorem.
- A complete solution gives the mathematical result and explains what it means. Check that it is possible in the stated context.

■ Use this in your course

- Current first-assessment-2021 Analysis and Approaches HL. This is authored concept support; the full guide is needed to certify every objective.
- Give the method before the final answer, and use the paper's calculator and formula rules. Review a wrong answer by locating the first invalid step.

Key:

The number obtained by reversing the sign of the imaginary part. Choose the relationship, show the method, check its assumptions and interpret the result.

2 Practice

2.1 Find $|3+4i|$. Show the relation, substitution and units. [2]

2.2 Find the real part of $(3+4i)(3-4i)$. Show the relation, substitution and units. [2]

2.3 Find the real part of $1/(3+4i)$. Show the relation, substitution and units. [2]

2.4 Decide whether this claim is true and justify: For $a+bi$, its modulus is always $a+b$. [2]

2.5 Which description correctly defines complex conjugate? Choose one answer.

- **A** The number obtained by reversing the sign of the imaginary part.
 - **B** A numerical answer that needs no assumptions or interpretation
 - **C** A rule that remains valid even when its required conditions fail
- [1]

3 Exam-style questions

3.1 Find all complex roots of $z^2 = -8i$ in Cartesian form and verify them. [4]

Solutions

2.1 Modulus= $\sqrt{(3^2+4^2)}=5$.

2.2 Conjugate multiplication gives $3^2+4^2=25$, with imaginary parts cancelling.

2.3 Multiply numerator and denominator by 3-4i: $1/(3+4i)=(3-4i)/25$. Its real part is $3/25=0.12$.

2.4 False. The square root of a sum is not the sum of the square roots. An inverse tangent alone may return the wrong quadrant. Do not confuse modulus with squared modulus.

2.5 A The number obtained by reversing the sign of the imaginary part.

3.1 Polar modulus 8 and argument $-\pi/2$ give roots modulus $\sqrt{8}$, arguments $-\pi/4$ and $3\pi/4$: $2-2i$ and $-2+2i$. Squaring each gives $-8i$. Both roots are needed.