

1.1 Exact arithmetic and estimation

Name: _____ Class: _____ Date: _____ Total: 13 marks

KEY WORDS prime factor

Objective

Exact arithmetic and estimation. Supported HL concept preparation; original classroom practice, not an official IB paper or assessed learner work.

1 Key knowledge and worked examples

■ Can we pack without leftovers?

- A supplier packs 72 pencils and 90 pens into identical gift bags. How can we avoid leftovers?
- This lesson studies **prime factor** : A prime number that divides the integer exactly.

■ Choose the mathematical structure

- Prime factors reveal shared structure. Use the smallest common prime powers for the HCF and the largest for the LCM. Estimate before calculating; use brackets to preserve the order of operations.
- State the allowed inputs and units before calculating. An equation should express the relationship, not just record a calculator entry.

■ Work through a checked case

$$a = \prod p_i^{\alpha_i}, \quad b = \prod p_i^{\beta_i}, \quad \text{HCF}(a, b) = \prod p_i^{\min(\alpha_i, \beta_i)}$$

- Check the result against the starting quantities. Substitute into the original relation, or compare the graph and numerical answer where appropriate.

Example:

$72=2^3 \times 3^2$ and $90=2 \times 3^2 \times 5$. Their HCF is $2 \times 9=18$. Make 18 bags with 4 pencils and 5 pens each. Their LCM is $2^3 \times 3^2 \times 5=360$.

$$72 = 2^3 \times 3^2$$

$$90 = 2 \times 3^2 \times 5$$

$$\text{Shared smallest powers: } 2 \times 3^2 = 18$$

■ Test a tempting shortcut

- The HCF divides both numbers; the LCM is a multiple of both. They answer different questions. A decimal estimate is not an exact fraction.
- When a shortcut fails, identify the assumption it breaks. Keep an exact value until the requested final rounding.

Warn:

The HCF of two positive integers is always larger than either integer. This claim is false. Explain which definition or assumption it violates.

■ Interpret a new situation

- For a non-calculator paper, keep fractions exact and show cancellation. For a calculator paper, enter the full expression and compare with your estimate.
- A complete solution gives the mathematical result and explains what it means. Check that it is possible in the stated context.

■ Use this in your course

- Current first-assessment-2021 Analysis and Approaches HL. This is authored concept support; the full guide is needed to certify every objective.
- Give the method before the final answer, and use the paper's calculator and formula rules. Review a wrong answer by locating the first invalid step.

Key:

A prime number that divides the integer exactly. Choose the relationship, show the method, check its assumptions and interpret the result.

2 Practice

2.1 Find the HCF of 48 and 60. Show the relation, substitution and units. [2]

2.2 Find the LCM of 8 and 12. Show the relation, substitution and units. [2]

2.3 Work out $3/4 + 5/6$ as a decimal. Show the relation, substitution and units. [2]

2.4 Decide whether this claim is true and justify: The HCF of two positive integers is always larger than either integer. [2]

2.5 Which description correctly defines prime factor? Choose one answer.

- **A** A prime number that divides the integer exactly.
- **B** A numerical answer that needs no assumptions or interpretation
- **C** A rule that remains valid even when its required conditions fail

[1]

3 Exam-style questions

3.1 A supplier has 84 pencils and 126 pens. Find the greatest number of identical bags without leftovers and the contents of each; then explain why the LCM answers a different question. [4]

Solutions

2.1 $48=2^4 \times 3$ and $60=2^2 \times 3 \times 5$. The shared smallest powers give $\text{HCF}=2^2 \times 3=12$.

2.2 $8=2^3$ and $12=2^2 \times 3$. The largest prime powers give $\text{LCM}=2^3 \times 3=24$.

2.3 Use denominator 12: $3/4+5/6=9/12+10/12=19/12$.

2.4 False. The HCF divides both numbers; the LCM is a multiple of both. They answer different questions. A decimal estimate is not an exact fraction.

2.5 A A prime number that divides the integer exactly.

3.1 $\text{HCF}=42$, each bag 2 pencils and 3 pens. $\text{LCM}=252$ is a common multiple, useful for matching repeated quantities, not the greatest number of bags dividing both stocks.