

Exercise walk-through

1.5

Complex numbers and roots

International Baccalaureate · IB Diploma · Mathematics: Analysis and Approaches

1.5 Complex numbers and roots

Name: _____ Class: _____ Date: _____ **Total: 13 marks**

KEY WORDS complex conjugate

Objective

Complex numbers and roots. Supported HL concept preparation; original classroom practice, not an official IB paper or assessed learner work.

1 Key knowledge and worked examples

■ What if the root is not real?

- The equation $x^2+1=0$ has no real roots. Extending the number system lets us represent and calculate with both roots.
- This lesson studies **complex conjugate** : The number obtained by reversing the sign of the imaginary part.

■ Choose the mathematical structure

- Write $z=a+bi$ with $i^2=-1$. The conjugate is $a-bi$ and z times its conjugate= a^2+b^2 . The modulus is $\sqrt{a^2+b^2}$; the argument needs the correct quadrant. Conjugate roots occur for polynomials with real coefficients.
 - State the allowed inputs and units before calculating. An equation should express the relationship, not just record a calculator entry.
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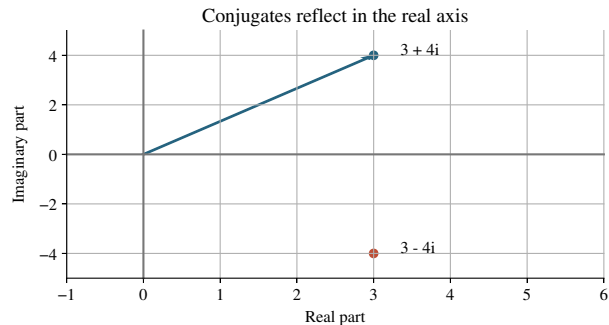
■ Work through a checked case

$$z = a + bi, \quad |z| = \sqrt{a^2 + b^2}, \quad z\bar{z} = a^2 + b^2$$

- Check the result against the starting quantities. Substitute into the original relation, or compare the graph and numerical answer where appropriate.

Example:

For $z=3+4i$, $|z|=5$ and z conjugate $=3-4i$. The reciprocal is $(3-4i)/25$. For $x^2-6x+13=0$, the roots are $3\pm 2i$. Multiplying by the conjugate makes a complex denominator real.



■ **Test a tempting shortcut**

- The square root of a sum is not the sum of the square roots. An inverse tangent alone may return the wrong quadrant. Do not confuse modulus with squared modulus.
- When a shortcut fails, identify the assumption it breaks. Keep an exact value until the requested final rounding.

Warn:

For $a+bi$, its modulus is always $a+b$. This claim is false. Explain which definition or assumption it violates.

■ **Interpret a new situation**

- Draw the real axis horizontally and imaginary axis vertically. For products, moduli multiply and arguments add; powers extend this geometric pattern through de Moivre's theorem.
 - A complete solution gives the mathematical result and explains what it means. Check that it is possible in the stated context.
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■ **Use this in your course**

- Current first-assessment-2021 Analysis and Approaches HL. This is authored concept support; the full guide is needed to certify every objective.
- Give the method before the final answer, and use the paper's calculator and formula rules. Review a wrong answer by locating the first invalid step.

Key:

The number obtained by reversing the sign of the imaginary part. Choose the relationship, show the method, check its assumptions and interpret the result.

Exercises 2.1·2.3

2.1 Find $|3+4i|$. Show the relation, substitution and units. [2]

2.2 Find the real part of $(3+4i)(3-4i)$. Show the relation, substitution and units. [2]

2.3 Find the real part of $1/(3+4i)$. Show the relation, substitution and units. [2]

Exercises 2.1·2.3

2.1 Find $|3+4i|$. Show the relation, substitution and units. [2]

2.1 Modulus= $\sqrt{(3^2+4^2)}=5$.

2.2 Find the real part of $(3+4i)(3-4i)$. Show the relation, substitution and units. [2]

2.2 Conjugate multiplication gives $3^2+4^2=25$, with imaginary parts cancelling.

2.3 Find the real part of $1/(3+4i)$. Show the relation, substitution and units. [2]

2.3 Multiply numerator and denominator by $3-4i$: $1/(3+4i)=(3-4i)/25$. Its real part is $3/25=0.12$.

Exercises 2.4·3.1

2.4 Decide whether this claim is true and justify: For $a+bi$, its modulus is always $a+b$. [2]

2.5 Which description correctly defines complex conjugate? Choose one answer.

- **A** The number obtained by reversing the sign of the imaginary part.
- **B** A numerical answer that needs no assumptions or interpretation
- **C** A rule that remains valid even when its required conditions fail [1]

3 Exam-style questions

3.1 Find all complex roots of $z^2 = -8i$ in Cartesian form and verify them. [4]

Exercises 2.4·3.1

2.4 Decide whether this claim is true and justify: For $a+bi$, its modulus is always $a+b$. [2]

2.4 False. The square root of a sum is not the sum of the square roots. An inverse tangent alone may return the wrong quadrant. Do not confuse modulus with squared modulus.

2.5 Which description correctly defines complex conjugate? Choose one answer.

- **A** The number obtained by reversing the sign of the imaginary part.
- **B** A numerical answer that needs no assumptions or interpretation
- **C** A rule that remains valid even when its required conditions fail [1]

3 Exam-style questions

2.5 A The number obtained by reversing the sign of the imaginary part.

3.1 Find all complex roots of $z^2 = -8i$ in Cartesian form and verify them. [4]

3.1 Polar modulus 8 and argument $-\pi/2$ give roots modulus $\sqrt{8}$, arguments $-\pi/4$ and $3\pi/4$: $2-2i$ and $-2+2i$. Squaring each gives $-8i$. Both roots are needed.