

23.1 Uncertainty, gradients and model testing

Name: _____ Class: _____ Date: _____

Total: 10 marks

KEY WORDS systematic error • uncertainty

Objective

Uncertainty, gradients and model testing. Supported HL concept preparation; original classroom practice, not an official IB paper or assessed learner work.

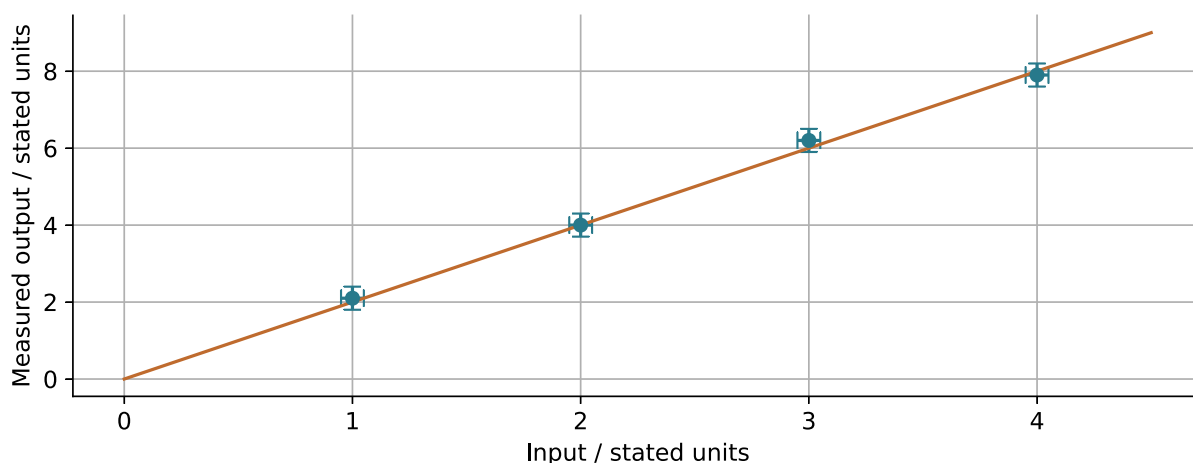
1 Key knowledge and worked examples

■ What would explain this observation?

- A line passing near every data point is useful, but its gradient can still be uncertain. A graph is evidence for a model within the measurement range.
- Start with a prediction. State the quantities or features you would compare, then decide what evidence could distinguish two explanations.

■ Build the model

- Random variation makes repeated readings differ. **Systematic error** shifts results consistently. Absolute **uncertainty** has the measured unit; relative or percentage uncertainty compares uncertainty with the measured value.
- uncertainty: A quantified limitation on a measured result; systematic error: A consistent measurement bias.



■ Choose evidence that can test it

- For a product or quotient, adding fractional uncertainties is a common maximum-uncertainty approximation. For a difference, add absolute uncertainties. A nonzero intercept can reveal an offset or an incomplete model.
- Show units on axes and choose a sensible scale. Plot uncertainty bars where justified, draw a best-fit line rather than joining every point, and estimate steepest and shallowest plausible gradients when the course method calls for them.

■ Work from known quantities

- State the known values and their units. Choose the relation because its assumptions fit this case, then rearrange before substitution.
- Known: length = 50.0 mm with uncertainty 1.0 mm. Percentage uncertainty = $\frac{\text{absolute uncertainty}}{\text{value}} \times 100 = \frac{1.0}{50.0} \times 100 = 2.0\%$. For a quotient of two independently measured quantities with maximum percentage uncertainties 2% and 3%, the summed maximum estimate is 5%.

Example:

A 40 cm reading has an absolute uncertainty of 1 cm. Find percentage uncertainty. Use the same sequence: known quantities → model → relation → substitution → unit and interpretation.

■ Check the conclusion and its limits

- Repeating readings reduces random uncertainty in a mean but does not automatically remove a zero error. Do not quote more decimal places than your measurement can support.
- Return to the original observation. Explain what the result supports, which conditions it assumes, and one way to test a competing explanation.

Warn:

Repeating a measurement always removes a calibration offset. This claim is false: Repeating readings reduces random uncertainty in a mean but does not automatically remove a zero error. Do not quote more decimal places than your measurement can support.

Key:

Uncertainty, gradients and model testing: For a product or quotient, adding fractional uncertainties is a common maximum-uncertainty approximation. For a difference, add absolute uncertainties. A nonzero intercept can reveal an offset or an incomplete model.

2 Practice

2.1 Which is most likely a systematic error? Choose one answer.

- **A** A balance reads 0.20 g when empty
- **B** Repeated values fluctuate either side of the mean
- **C** Random differences in reaction time

[1]

2.2 A 40 cm reading has an absolute uncertainty of 1 cm. Find percentage uncertainty. Show the relation, substitution and units.

[2]

2.3 Decide whether this claim is true and justify: Repeating a measurement always removes a calibration offset.

[2]

2.4 A quantified limitation on a measured result: write the technical term.

[1]

3 Exam-style questions

3.1 Lengths are (50 ± 1) mm and (20 ± 1) mm. Estimate the quotient and its maximum percentage uncertainty; distinguish this convention from independent standard-uncertainty propagation. [4]

Solutions

2.1 A For a product or quotient, adding fractional uncertainties is a common maximum-uncertainty approximation. For a difference, add absolute uncertainties. A nonzero intercept can reveal an offset or an incomplete model.

2.2 percentage uncertainty = $1/40 \times 100 = 2.5\%$.

2.3 False. Repeating readings reduces random uncertainty in a mean but does not automatically remove a zero error. Do not quote more decimal places than your measurement can support.

2.4 uncertainty. uncertainty means A quantified limitation on a measured result.

3.1 Quotient 2.5. Maximum estimate $2\% + 5\% = 7\%$, absolute $0.175 \approx 0.18$. Independent standard uncertainties instead combine in quadrature, about 5.39% ; declare which convention the data support.