

Thermal transport, calorimetry and expansion

GRE · GRE Subject Test · GRE Physics

Two wall layers in series carry the same steady heat rate, but their temperature drops need not be equal.

Prerequisites: 4.

- Combine conductive thermal resistances and distinguish heat rate from heat flux
- Balance sensible and latent heat with stated isolation and heat-capacity conditions
- Calculate free thermal expansion and constrained thermal stress with their limits

Add thermal resistances

For one-dimensional steady conduction through a uniform slab, Fourier's law gives heat flux $q_x = -\kappa dT/dx$. With constant conductivity κ , area A , length L and hot-to-cold temperature difference ΔT , heat rate is $\dot{Q} = \kappa A \Delta T / L$. Define thermal resistance

$R_{th} = L/(\kappa A)$, measured in K/W; temperature drop is $\dot{Q} R_{th}$. Series layers without internal sources carry the same $\dot{Q}$ and their resistances add. Parallel paths at the same endpoint temperatures have heat rates that add, so inverse resistances add. Heat flux is rate per area, W/m², and need not match between layers of different area. Interface contact resistance and heat leakage must be included if specified.

Balance sensible and latent energy

Heat capacity $C = dQ/dT$ depends on the thermodynamic path and is not the same as specific heat c per mass. For a material interval without a phase transition, $Q = \int mc(T)dT$, or $mc\Delta T$ when c is constant. In an isolated calorimeter, sum the energy changes of all objects, including the container when relevant, and set the sum to zero. At a phase transition at its equilibrium temperature, added energy can change phase fraction rather than temperature: $Q = m\ell$ uses latent heat ℓ in J/kg. First supply the sensible heat to reach the transition, then budget latent heat. Do not let a simple weighted-temperature average predict an impossible temperature when melting or freezing is part of the process.

Check expansion constraints

For a freely expanding rod and small temperature change, $\Delta L \approx \alpha L_0 \Delta T$ with linear expansion coefficient α . For an isotropic solid with small strain, each dimension acquires factor $1 + \alpha \Delta T$, so $\Delta A/A \approx 2\alpha \Delta T$ and $\Delta V/V \approx 3\alpha \Delta T$. A hole expands with the surrounding material as if its missing region had expanded too. These relations assume a nearly constant coefficient and no mechanical constraint; anisotropic crystals need directional coefficients. If an elastic rod is prevented from changing length, its

mechanical strain cancels thermal strain. With tensile stress positive, $\sigma \approx -Y\alpha \Delta T$ during heating, where Y is Young's modulus. This small-strain estimate requires elastic response without yielding or buckling.

Choose the transport model

Conduction transfers energy through a temperature gradient, convection transports it with moving matter, and thermal radiation can cross a vacuum. For a surface at T facing large surroundings at T_{env} , a simple grey-body model gives net radiative rate $\varepsilon \sigma_{SB} A(T^4 - T_{env}^4)$. Use absolute kelvin in these fourth powers, rather than Celsius values. A lumped body whose internal temperature remains nearly uniform can obey $C dT/dt = -hA(T - T_{env})$ under Newton cooling with constant h . Then the temperature excess decays with $\tau = C/(hA)$, rather than the entire Celsius or kelvin temperature decaying toward zero. This approximation requires sufficiently small internal gradients; a conduction-limited body needs a spatial temperature model.

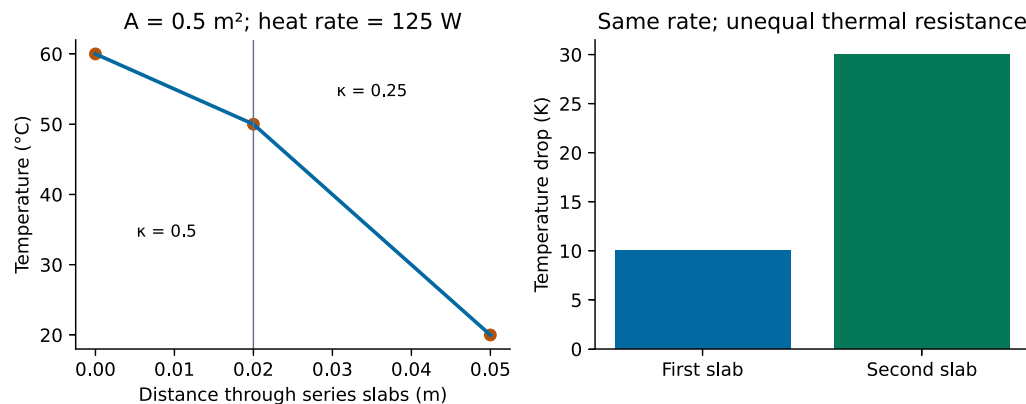
Worked method

Two slabs in series carry the same steady heat rate. Let resistances be 0.08 and 0.24 K/W. Boundary temperatures are 60 and 20 degrees Celsius.

$$\dot{Q} = \Delta T / (R_1 + R_2) = (40 \text{ K}) / (0.32 \text{ K/W}) = 125 \text{ W}.$$

$$\Delta T_1 = \dot{Q} R_1 = (125 \text{ W})(0.08 \text{ K/W}) = 10 \text{ K}.$$

The interface is 50 degrees Celsius when slab 1 touches the hot side. Heat rate has watt units; flux divides it by area.



Thermal transport, calorimetry and expansion: original GRE teaching diagram.

Check conditions and vocabulary

Keep W separate from W/m^2 , include the calorimeter or latent energy when needed, and do not use free-expansion length together with constrained-stress assumptions.

thermal resistance: Temperature difference per steady heat-transfer rate, measured in K/W.

latent heat: Energy transferred during a phase change at its transition temperature; specific latent heat is per mass.