

# Polarisation, analyser chains and phase

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Inserting a third polariser between crossed filters can increase the transmitted intensity, even though every filter removes energy.

Prerequisites: 3.

- Calculate successive ideal analyser transmissions from the immediate input state
- Distinguish linear, circular and elliptical field motion using relative phase
- Use Brewster incidence with refractive-index and plane-of-incidence conventions

## Project the electric field

For propagation along  $z$ , a transverse electric field can have  $x$  and  $y$  components. Linear polarisation means the field oscillates along one fixed line; an ideal analyser transmits the projection along its axis. Field amplitude becomes  $E \cos\theta$ , so intensity becomes  $I \cos^2\theta$ . Here  $\theta$  is between the incoming polarisation and that analyser, and  $I$  is the intensity immediately before it. An unpolarised beam is a statistical mixture of transverse orientations; an ideal first polariser passes half its mean intensity. The outgoing beam is then linearly polarised along the filter axis. Do not apply a new factor of one half at every later filter, or use one angle to the original source for the whole chain.

## Update each analyser input

After each analyser, update both the intensity and the polarisation direction. For an initially unpolarised beam and ideal axes  $\alpha_1, \alpha_2, \alpha_3$ , the final intensity is  $(I_0/2)\cos^2(\alpha_2-\alpha_1)\cos^2(\alpha_3-\alpha_2)$ . Two crossed filters transmit zero in this model; a middle oblique axis changes the direction before the final projection. This increase relative to the crossed pair does not create energy: every step still has transmission between zero and one. Real filters have absorption, imperfect extinction and wavelength dependence. Use the ideal law only when those losses are excluded or separately specified.

## Track transverse phase

Write  $E_x = A \cos\omega t$  and  $E_y = B \cos(\omega t + \delta)$  at a fixed point. Equal or opposite phases give a line, including a line at an oblique angle; equal nonzero amplitudes and a quarter-cycle phase difference give a circle. Unequal amplitudes at quarter-cycle phase give an ellipse. A quarter-wave plate adds a relative phase of  $\pi/2$  between its principal axes under its design conditions. A linear input at  $45^\circ$  supplies equal components, so the outgoing field can be circular. A linear input along a principal axis has only one component and remains linear. For circular input, every ideal linear analyser passes half the intensity; handedness requires a declared viewing direction

and phase convention.

## Apply Brewster geometry

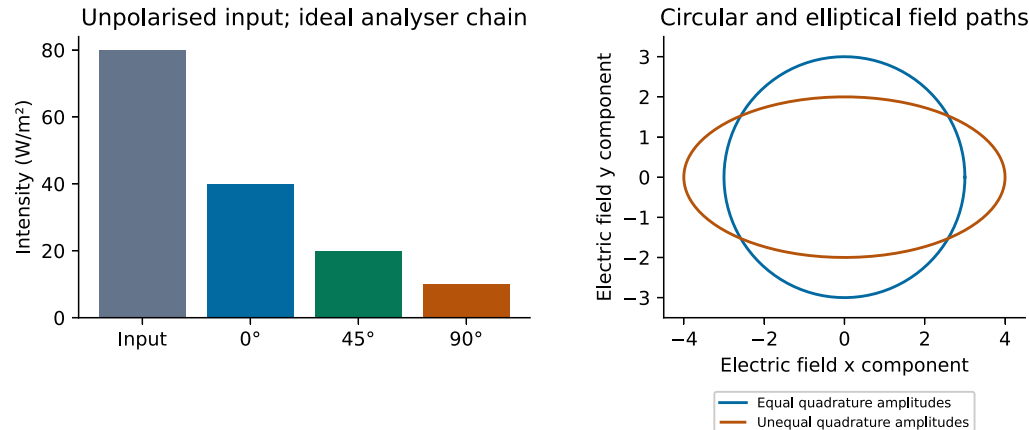
For incidence from transparent nonmagnetic medium  $n_1$  into  $n_2$ , Brewster's angle measured from the normal satisfies  $\tan\theta_B = n_2/n_1$ . At that angle, the reflected  $p$  component, whose electric field lies in the plane of incidence, vanishes in the ideal dielectric model. Reflected light from unpolarised input is then  $s$  polarised, perpendicular to that plane. Snell's law gives a refracted angle complementary to  $\theta_B$ . This is distinct from the critical-angle condition  $\sin\theta_c = n_2/n_1$ , which needs  $n_1 > n_2$  and concerns total internal reflection. Brewster reflection does not imply that the entire incident intensity is reflected or that the transmitted beam is completely polarised.

## Worked method

An ideal first polariser transmits half of unpolarised incident intensity  $I_0$ . Each later analyser uses the angle from the immediately preceding axis. For axes  $0, 30$  and  $90$  degrees,

$$I_f = \frac{1}{2}I_0 \cos^2(30^\circ) \cos^2(60^\circ) = 3I_0/32.$$

The field after each filter points along that filter's axis. Using only the first and last axes would predict zero incorrectly.



*Polarisation, analyser chains and phase: original GRE teaching diagram.*

## Check conditions and vocabulary

Malus intensity uses  $\cos^2$  of the angle to the immediately preceding polarisation; a phase plate changes relative phase, while a polariser removes a field component.

**Malus's law** : Ideal linear-analyser intensity law  $I_{\text{out}} = I_{\text{in}} \cos^2\theta$  for linearly polarised input.

**Brewster angle** : Incidence angle at which the reflected p component vanishes for the ideal dielectric interface.