

Polarisation, magnetisation and material fields

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Polarisation, magnetisation and material fields

Putting a dielectric into a capacitor changes the field differently depending on whether its battery remains connected.

Prerequisites: 13, 15.

- Relate free and bound charge to D , E and polarisation
- Compare fixed-charge and fixed-voltage dielectric changes
- Use magnetic constitutive response and free-current boundary conditions

Separate free and bound charge

Electric polarisation P is electric dipole moment per unit volume, measured in C/m^2 . Define $D = \epsilon_0 E + P$ so that $\nabla \cdot D = \rho_{\text{free}}$; the total charge, including bound charge, still appears in $\nabla \cdot E = \rho_{\text{total}}/\epsilon_0$. Bound volume charge is $\rho_{\text{b}} = -\nabla \cdot P$ and bound surface charge is $\sigma_{\text{b}} = P \cdot \mathbf{n}$, where $\mathbf{n}$ points outward from the material. Uniform P therefore gives no bound charge in the bulk but can give surface charges of opposite sign. A dielectric is not an ideal metal: polarisation need not make its internal electric field zero.

Apply the material relation

For a linear isotropic dielectric, $P = \epsilon_0 \chi_e E$ and $D = \epsilon E$ with $\epsilon = \epsilon_0(1 + \chi_e) = \epsilon_0 \epsilon_r$. These simple scalar relations assume the response is linear and ignore anisotropy, strong dispersion and nonlinear effects. In a fully filled, large parallel-plate capacitor with negligible edge effects, D normal to the plates equals the free surface-charge density. If σ_{free} stays fixed, increasing ϵ reduces $E = \sigma_{\text{free}}/\epsilon$. The bound charges oppose the applied field; their magnitude is not automatically equal to the free plate charge.

Hold charge or voltage fixed

For plate area A and spacing d , $C = \epsilon A/d$. Disconnecting the battery fixes free charge Q : $V = Q/C$ falls and stored energy $U = Q^2/(2C)$ falls when a dielectric increases C . Leaving an ideal voltage source connected fixes V : $Q = CV$ and stored energy $U = CV^2/2$ both rise. The source supplies energy and moving the dielectric can involve mechanical work; compare the stated electrical energy quantity rather than assuming the capacitor alone is an isolated system. At a material interface $\mathbf{n} \cdot (D_2 - D_1) = \sigma_{\text{free}}$, whereas static tangential E is continuous. Normal E generally changes when permittivity changes.

Distinguish magnetic fields

Magnetisation M is magnetic dipole moment per unit volume, measured in A/m . In SI, $B = \mu_0(H + M)$; for a linear isotropic response $M = \chi_m H$ and $B = \mu_0(1 + \chi_m)H$. B and H have different units and roles. A long uniform solenoid has $H \approx nI$ when end and demagnetising effects are negligible; material response then changes B . Free surface current sets $\mathbf{n} \times (H_2 - H_1) = K_{\text{free}}$, and normal B remains continuous. Ferromagnetic hysteresis and saturation cannot be represented by one constant χ_m over every field.

Worked method

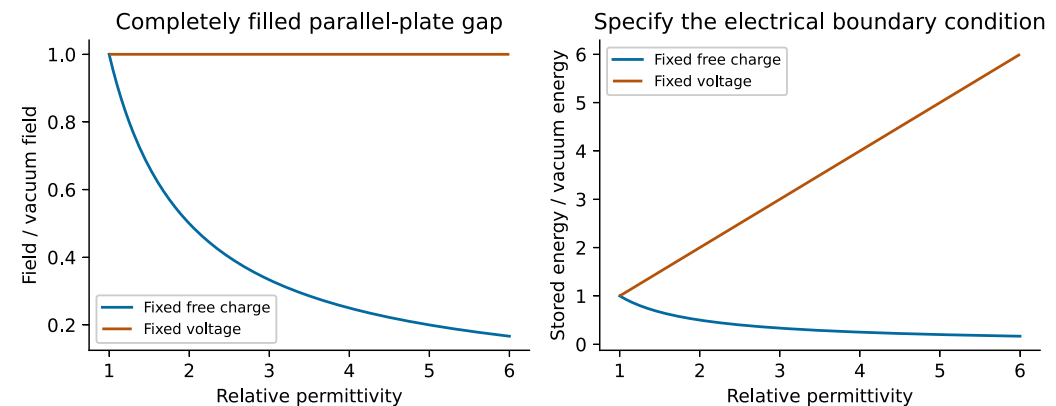
A capacitor is disconnected before inserting a linear dielectric with relative permittivity 4. Free charge stays fixed; capacitance becomes four times its original value.

$$V' = Q/(4C) = V/4, \quad U' = Q^2/(2 \times 4C) = U/4.$$

If an ideal battery stays connected instead, V is fixed:

$$Q' = 4CV = 4Q, \quad U' = \frac{1}{2}(4C)V^2 = 4U.$$

The source exchanges energy. Do not compare isolated charge and fixed voltage as the same experiment.



Polarisation, magnetisation and material fields: original GRE teaching diagram.

Check conditions and vocabulary

Use free charge in the D equation and total charge in the E equation. Do not confuse fixed Q with fixed V , or identify B numerically with H .

electric polarisation: Electric dipole moment per unit volume, P , contributing bound charge.

magnetisation: Magnetic dipole moment per unit volume, M , entering $B = \mu_0(H + M)$.