

Counting statistics, uncertainty and dimensional models

GRE · GRE Subject Test · GRE Physics

A detector with ten percent efficiency does not register exactly ten photons out of every hundred. The count fluctuates even with stable efficiency.

Prerequisites: 47.

- Use binomial and Poisson count means and fluctuations
- Propagate small uncertainties with stated correlation assumptions
- Solve dimensional exponent constraints and check a model's units

Choose the system and model

For N independent incident particles each detected with probability p , count X is binomial: mean Np and variance $Np(1-p)$. The standard deviation is $\sqrt{Np(1-p)}$, not the variance itself. With large N and small p , a Poisson approximation

has mean $\lambda=Np$ and variance λ , so deviation $\sqrt{\lambda}$. For $N=200$ and $p=0.05$, the exact mean is 10 and deviation $\sqrt{9.5}\approx 3.08$, while Poisson gives $\sqrt{10}\approx 3.16$. Neither model promises a fixed count. Correlated detections or dead time can invalidate independence.

Use the governing relation

For a smooth measured function $y(x_1, \dots)$, linearise changes using its partial derivatives. Independent small standard uncertainties combine in quadrature: $\sigma_y^2 \approx \sum (\partial y / \partial x_i)^2 \sigma_{x_i}^2$. Correlated inputs require covariance cross terms. For $y = x^a$, fractional standard uncertainty is approximately $|a| \sigma_x / |x|$. Thus kinetic energy $K = \frac{1}{2}mv^2$ with negligible mass uncertainty has fractional uncertainty twice that of speed. With independent mass uncertainty, combine $(\sigma_m/m)^2 + (2\sigma_v/v)^2$. These approximations need small errors and a suitable local linear model.

Apply the conditions

Random scatter measures precision; a common calibration bias affects accuracy and does not disappear by averaging repeats. For independent repeated readings, the standard uncertainty of the mean falls as $1/\sqrt{n}$, but shared systematic error does not. The uncertainty of a physical spread and uncertainty of its estimated mean are different quantities. State whether a quoted percentage is a standard uncertainty, confidence interval or worst-case bound. Summing absolute contributions is a conservative bound, not the independent-standard-error quadrature rule.

Check the conclusion

Dimensional analysis equates powers of mass, length and time, rather than numerical sizes. For a Planck-length form $G^a \hbar^b c^d$, dimensions are $[G]=L^3/(MT^2)$, $[\hbar]=ML^2/T$ and $[c]=L/T$. Requiring length gives $-a+b=0$, $3a+2b+d=1$ and $-2a-b-d=0$. Hence $a=b=1/2$ and $d=-3/2$, so length is $\sqrt{G\hbar/c^3}$. Dimensional analysis cannot determine an arbitrary dimensionless coefficient or prove that the chosen constants are physically sufficient. Reject a formula with wrong units before inserting numbers.

Worked method

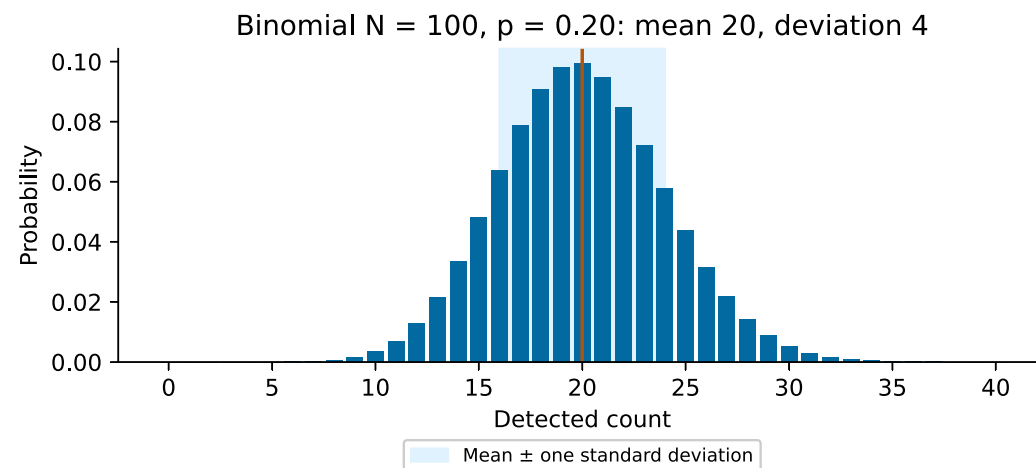
For independent small standard uncertainties in $K = mv^2/2$, differentiate first.

$$\left(\frac{\sigma_K}{K}\right)^2 = \left(\frac{\sigma_m}{m}\right)^2 + \left(2\frac{\sigma_v}{v}\right)^2.$$

With relative uncertainties 3% in mass and 2% in speed,

$$\sigma_K/K = \sqrt{0.03^2 + (2 \times 0.02)^2} = 0.05.$$

A worst-case bound would use a sum, not this quadrature. Correlations require covariance terms.



Counting statistics, uncertainty and dimensional models: original GRE teaching diagram.

Check conditions and vocabulary

Do not confuse mean count with guaranteed count, variance with standard deviation, or a random standard error with a calibration bias. Dimensional consistency is necessary but not sufficient.

standard uncertainty: Uncertainty expressed as a standard deviation under a stated measurement model.

Poisson approximation: Rare independent-event count model with variance equal to its mean.