

Maxwell waves, energy flow and boundary conditions

GRE · GRE Subject Test · GRE Physics

A charged capacitor can change its electric field across a gap even though no conduction current crosses the gap.

Prerequisites: 13, 29, 47.

- Use displacement current and material wave-speed relations
- Determine wave-field directions and radiated energy flow
- Apply Maxwell boundary conditions without confusing field components

Choose the system and model

Ampere–Maxwell law in vacuum is $\oint \mathbf{B} \cdot d\mathbf{l} = \mu_0(I_{\text{conduction}} + \epsilon_0 d\Phi_E/dt)$. The added displacement-current term depends on the time rate of electric flux through the chosen surface. It ensures consistent results for a loop whose spanning surface either crosses a capacitor wire or passes between the plates. Displacement current is not magnetic flux or an integral over earlier electric flux. For a uniform plate field, the gap contribution is $\epsilon_0 A dE/dt$; signs follow the oriented surface.

Use the governing relation

In a homogeneous, linear, lossless dielectric with permeability μ and permittivity ϵ , Maxwell equations give wave speed $v = 1/\sqrt{\mu\epsilon}$ and refractive index $n = c/v = \sqrt{\mu_r\epsilon_r}$. For nonmagnetic material $\mu_r \approx 1$, so $v = c/\sqrt{\epsilon_r}$. Use the stated frequency-dependent material constants when dispersion matters; a static dielectric constant need not describe every optical frequency. Vacuum waves have $E/B = c$; in this simple medium the relation is $E/B = v$.

Apply the conditions

For a plane wave travelling along unit vector $\mathbf{n}$, $\mathbf{B} = (\mathbf{n} \times \mathbf{E})/v$. Both fields are perpendicular to propagation and to each other. Phase $kz - \omega t$ propagates toward $+z$, while $kz + \omega t$ propagates toward $-z$. If $\mathbf{E}$ is along $x+y$ and propagation is $+z$, $\mathbf{B}$ is along $-x+y$, because $z \times x = y$ and $z \times y = -x$. The Poynting vector $\mathbf{S} = \mathbf{E} \times \mathbf{H}$ ($\mathbf{E} \times \mathbf{B}/\mu$ in this medium) points along energy transport. In the radiation zone of an accelerating charge, energy flux is outward from the source, not necessarily in the instantaneous direction of charge motion.

Check the conclusion

Maxwell's divergence equation $\nabla \cdot \mathbf{B} = 0$ gives continuity of the normal $\mathbf{B}$ component across an interface: a thin pillbox has no magnetic charge inside. The tangential $\mathbf{H}$ jump is related to surface current; it need not vanish. Under an ideal superconducting Meissner-state model with zero interior $\mathbf{B}$, the exterior normal component at the boundary must therefore be zero, leaving any nonzero exterior $\mathbf{B}$ tangent to the surface. This conclusion is conditional on the stated zero-interior model; it does not follow by assuming all exterior field is zero. For electrostatics, normal $\mathbf{D}$ can instead jump by free surface charge.

Worked method

A plane wave travels along positive z in a linear lossless nonmagnetic dielectric. With relative permittivity 9,

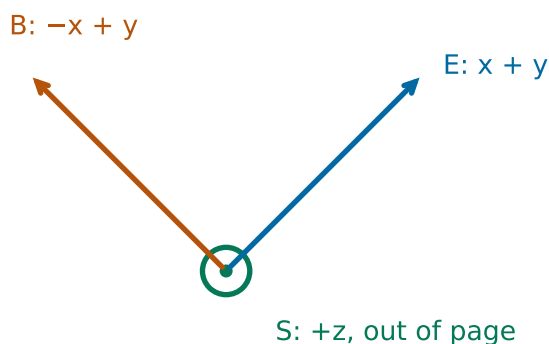
$$v = c/\sqrt{\epsilon_r} = (3.00 \times 10^8 \text{ m/s})/3 = 1.00 \times 10^8 \text{ m/s}.$$

If $\mathbf{E}$ points along positive x , $\mathbf{B} = \hat{\mathbf{z}} \times \mathbf{E}/v$ points along positive y . For $E_0 = 200 \text{ V/m}$,

$$B_0 = E_0/v = (200 \text{ V/m})/(1.00 \times 10^8 \text{ m/s}) = 2.00 \mu\text{T}.$$

$\mathbf{E} \times \mathbf{H}$ points along propagation. Check that material constants apply at the wave frequency.

$\mathbf{E} \times \mathbf{B}$ points in the propagation direction



Maxwell waves, energy flow and boundary conditions: original GRE teaching diagram.

Check conditions and vocabulary

Displacement current uses changing electric flux. Zero interior B fixes the exterior normal component, not every exterior component. Distinguish wave propagation from particle motion.

displacement current: Electric-flux time-derivative contribution in Ampere–Maxwell law.

Poynting vector: Electromagnetic energy flux per unit area and time.