

Electrostatic superposition, flux and conductors

GRE · GRE Subject Test · GRE Physics

A charged conductor can have zero field inside its material while its potential remains nonzero.

Prerequisites: 2, 47.

- Add Coulomb forces as vectors using the actual geometry
- Compute flux from symmetry or solid angle with a stated orientation
- Find conductor fields and potentials using induced surface charges

Choose the system and model

For a point charge Q at position $\mathbf{r}_0$, $E(\mathbf{r}) = Q(\mathbf{r} - \mathbf{r}_0) / (4\pi\epsilon_0 |\mathbf{r} - \mathbf{r}_0|^3)$. Superpose vectors, not field magnitudes. For two positive charges attracting an electron, forces in the same direction add; opposite directions subtract; perpendicular components combine by Pythagoras. Define the observation point and the direction of each force first. Potential is the scalar sum $V = \sum Q / (4\pi\epsilon_0 r)$ with zero at infinity. A zero potential does not generally mean a zero field, since $E = -\nabla V$.

Use the governing relation

Gauss's law is the closed-surface integral $\oint \mathbf{E} \cdot d\mathbf{A} = Q_{\text{enclosed}} / \epsilon_0$. It determines a local field simply only when symmetry makes the normal field uniform or makes other flux contributions zero. A charge near an infinite plane sends half its total flux through that plane in magnitude: the plane subtends solid angle 2π out of 4π . The result is independent of distance and lateral position, but its sign depends on the chosen plane normal and charge sign. An open plane does not enclose a charge; using closed-surface wording for it is incorrect.

Apply the conditions

In electrostatic equilibrium, the electric field inside conducting material is zero and the conductor has constant potential. Place a charge Q at the centre of a conducting spherical shell with inner radius a , outer radius b and shell net charge q . A Gaussian surface inside the material requires inner-surface charge $-Q$, so the outer surface has $q + Q$. Spherical symmetry makes the external field that of total $Q + q$ at the centre. The material's potential, zero at infinity, is $(Q + q) / (4\pi\epsilon_0 b)$, independent of the material observation radius.

Check the conclusion

In the cavity of that centred-charge shell, the field is $Q / (4\pi\epsilon_0 r^2)$ radially for $0 < r < a$. Potential includes both the point-charge contribution and constant shell contributions; continuity holds across each surface even though the normal field jumps at a surface charge. Keep cavity, conductor material and exterior separate. An off-centre charge still induces total inner charge $-Q$, but its cavity field is no longer the simple centred radial field. Zero interior conductor field follows equilibrium, not an assumption that every surface charge distribution is uniform.

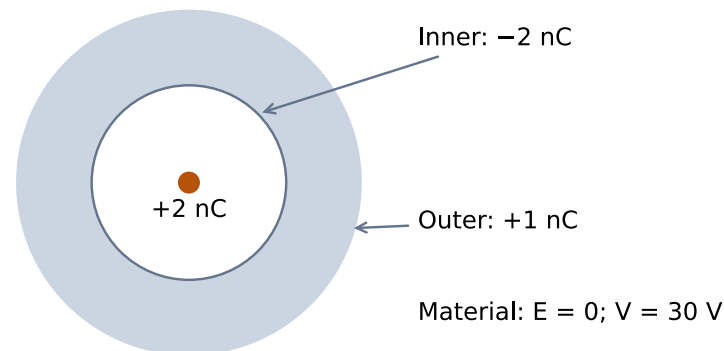
Worked method

A centred point charge Q lies in a conducting spherical shell of net charge q . Electrostatic equilibrium makes the field zero inside the metal. A Gaussian surface in the metal encloses zero net charge, so $Q_{\text{inner}} = -Q$. Charge conservation gives $Q_{\text{outer}} = q + Q$. With outer radius b and zero potential at infinity,

$$V_{\text{metal}} = \frac{q + Q}{4\pi\epsilon_0 b} = \frac{(9.0 \times 10^9 \text{ N m}^2/\text{C}^2)(1.0 \times 10^{-9} \text{ C})}{0.30 \text{ m}} = 30 \text{ V}.$$

Here $Q = +2 \text{ nC}$ and $q = -1 \text{ nC}$. The cavity is not conducting material; its field need not vanish.

Shell net charge -1 nC ; outer radius 0.30 m



Electrostatic superposition, flux and conductors: original GRE teaching diagram.

Check conditions and vocabulary

Do not confuse the open-plane half-flux result with enclosed charge. A conductor's zero field fixes a constant potential, not necessarily zero potential.

solid angle: Angular area subtended by a surface, measured in steradians.

induced surface charge : Charge rearranged on a conductor to satisfy electrostatic equilibrium.