

Rotation, buoyancy and terminal-speed balances

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Turning a floating block upside down can change how much wood is submerged even though the total weight stays the same.

Prerequisites: 10.

- Use pivot torque and rotational inertia consistently
- Partition rolling kinetic energy with the no-slip constraint
- Balance buoyancy and drag for composite bodies

Choose the system and model

Torque about a fixed pivot is $\mathbf{r} \times \mathbf{F}$; its magnitude uses the perpendicular lever arm. For a uniform rod of length L pivoted at one end, gravity acts at its centre $L/2$ and the moment of inertia is $ML^2/3$. If its angle θ is measured from vertical, gravitational torque magnitude is $Mg(L/2)\sin\theta$, so angular-acceleration magnitude is $3g \sin\theta/(2L)$. Define the positive rotation direction before assigning a sign. Using the centre-of-mass inertia $ML^2/12$ without the parallel-axis correction gives a wrong acceleration.

Use the governing relation

An object rolling without slipping satisfies $v=R\omega$. Write total kinetic energy as $\frac{1}{2}Mv^2 + \frac{1}{2}I\omega^2$. With $I=\beta MR^2$, the rotational fraction is $\beta/(1+\beta)$; for a solid disk $\beta=1/2$ and the fraction is $1/3$. A hoop has $\beta=1$ and fraction $1/2$. Static friction can supply the torque required for rolling without dissipating energy at an instantaneously stationary contact on a fixed surface. A sliding object does not satisfy the no-slip relation automatically.

Apply the conditions

For a floating composite, buoyancy equals the weight of displaced fluid, summed over every submerged part. Let a wood block have volume V and a dense stone volume V_s . If the stone is above the water, only submerged wood contributes: $\rho_{\text{water}} g fV$ equals total weight. If the stone is attached below and fully submerged, its displaced volume reduces the wood's required submerged fraction to $f - V_s/V$. The stone does not cease to weigh anything; the water now supplies some of its support. Check whether the assumed orientation and flotation are physically possible.

Check the conclusion

Terminal speed means zero acceleration, not zero speed or zero force. With quadratic drag $C v^2$ and a downward weight Mg , neglecting buoyancy gives $vt=\text{sqrt}(Mg/C)$. Two otherwise identical balls share the drag coefficient C ; doubling mass then multiplies terminal speed by $\text{sqrt}(2)$. If buoyancy matters, replace Mg by $(M-\rho_{\text{fluid}} V)g$. A linear-drag model instead gives a different mass scaling. State the specified drag law and compare force balances, rather than transferring a formula between models.

Worked method

A body rolls without slipping on a fixed surface. The no-slip constraint is $v = R \omega$.

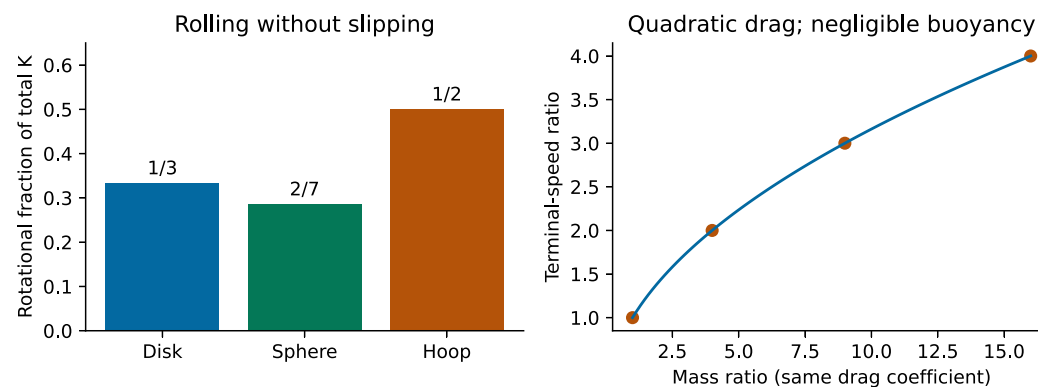
$$K = \frac{1}{2}Mv^2 + \frac{1}{2}I\omega^2 = \frac{1}{2}Mv^2(1 + \beta), \quad I = \beta MR^2.$$

For a solid disk, $\beta = 1/2$, so the rotational share is

$$\frac{K_{\text{rot}}}{K} = \frac{\beta}{1 + \beta} = \frac{1/2}{3/2} = \frac{1}{3}.$$

For a uniform rod pivoted at one end, use $I = ML^2/3$, not its centre-of-mass inertia. About this pivot $\tau = -Mg(L/2)\sin\theta$, for θ measured from downward vertical.

$$\ddot{\theta} = \tau/I = -3g \sin\theta/(2L).$$



Rotation, buoyancy and terminal-speed balances: original GRE teaching diagram.

Check conditions and vocabulary

Use the inertia about the chosen pivot. In flotation, include the stone's displaced volume as well as its weight. Terminal speed requires a specified drag law.

moment of inertia: Mass-weighted squared perpendicular distance from a specified rotation axis.

terminal speed: Steady speed at which opposing forces balance.