

Solutions

2.1 Answer: B. $p_{\text{exc}} = 1/(1 + e^x) = 1/(1 + 4) = 1/5$.

Compare the rejected options: $1/4$; $4/5$; $\ln 4$; $1/2$. Use the stated model and conditions.

2.2 Answer: C. All probabilities are unchanged. U and F shift by ε and Z gains $e^{-\beta\varepsilon}$, but entropy and heat capacity remain the same.

Compare the rejected options: Mean energy U ; Free energy F ; Every energy eigenvalue; Z itself. Use the stated model and conditions.

2.3 1. Mean $= 1$ J and second moment $= 2$ J², so variance $= 2 - 1^2 = 1$ J².

2.4 True. Energy approaches $\Delta/2$ and its temperature derivative tends to zero; a finite spectrum saturates.

3.1 Write $x = \beta\Delta$, $w = ge^{-x}$, $z = 1 + w$ and $Z = z^N$. $U = N\Delta w/(1 + w)$; $\text{Var}(E) = N\Delta^2 w/(1 + w)^2$. $C = Nk_B x^2 w/(1 + w)^2$. At high T every complete state is equally likely, so $S \rightarrow Nk_B \ln(1 + g)$. C tends to zero at low T and at high T . Award Z , U , variance/ C , entropy and limits. This is not a quantum-gas state-counting model.