

Solutions

2.1 C Adding $\dot{x}^2$ and $\dot{y}^2$ gives $l^2\dot{\theta}^2$.

Compare the rejected options: $ml^2 \sin^2\theta \dot{\theta}^2/2$; $ml^2 \cos^2\theta \dot{\theta}^2/2$; $mg l\theta^2/2$; $ml\dot{\theta}/2$. Use the stated model and conditions.

2.2 D $\partial L/\partial\theta=0$ gives $d(mr^2\dot{\theta})/dt=0$; radius and angular speed may change.

Compare the rejected options: θ is constant; $\dot{\theta}$ is constant; r is constant; Mechanical kinetic energy is constant. Use the stated model and conditions.

2.3 3. $V''=9$ and $\sqrt{9/1}=3$.

2.4 False. $\sin\theta\approx\theta$ requires radians; a degree value needs conversion.

3.1 Prescribed rotation gives $L = m(\dot{r}^2 + \Omega^2 r^2)/2$. Euler-Lagrange gives $\ddot{r} = \Omega^2 r$. The time-independent reduced L has conserved $\dot{r} \partial L/\partial \dot{r} - L = m(\dot{r}^2 - \Omega^2 r^2)/2$. Mechanical energy is $m(\dot{r}^2 + \Omega^2 r^2)/2$ and is not generally conserved: the rotation drive supplies work through the constraint. Award L , equation, conserved quantity, mechanical-energy distinction and drive. The bead's angular momentum is not conserved.