

21.1 Counting statistics, uncertainty and dimensional models

Name: _____ Class: _____ Date: _____ Total: 10 marks

KEY WORDS standard uncertainty • Poisson approximation

Objective

Use binomial and Poisson count means and fluctuations; Propagate small uncertainties with stated correlation assumptions; Solve dimensional exponent constraints and check a model’s units. Original public practice; not an ETS paper.

1 Key knowledge and worked examples

For N independent detection trials with probability p, binomial mean is Np and variance Np(1−p). Standard deviation is its square root. For large N and small p, Poisson mean/variance are λ=Np. Stable efficiency does not guarantee a fixed count; correlations or dead time can invalidate independence.

Linearised independent uncertainties combine as $\sigma_y^2 \approx \sum (\partial y / \partial x_i)^2 \sigma_{x_i}^2$. Correlations need covariance terms. A power $y = x^a$ has fractional uncertainty about $|a| \sigma_x / |x|$. For $K = \frac{1}{2}mv^2$, independent mass/speed contributions are combined as $\text{sqrt}[(\sigma_m/m)^2 + (2\sigma_v/v)^2]$. Use small standard errors, not unstated bounds.

Random scatter measures precision; calibration bias affects accuracy. Independent mean uncertainty falls as $1/\text{sqrt}(n)$, but common bias does not. Physical spread and uncertainty of its mean differ. State whether percentages are standard errors, confidence intervals or worst-case bounds.

For $G \hat{a} \hbar \hat{b} c \hat{d}$ to be a length, dimensional powers give $-a+b=0$, $3a+2b+d=1$ and $-2a-b-d=0$. Hence $a=b=1/2$, $d=-3/2$ and length $\text{sqrt}(G\hbar/c^3)$. Unit consistency cannot determine a dimensionless coefficient or validate physical assumptions.

■ Worked method

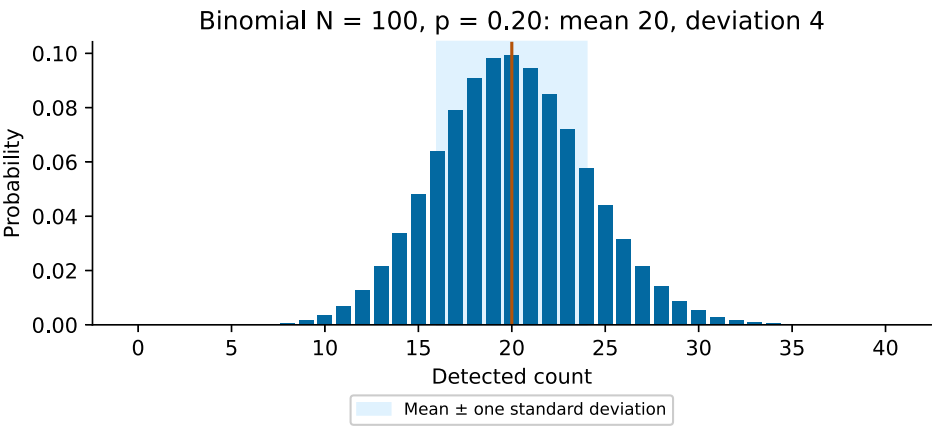
For independent small standard uncertainties in $K = mv^2/2$, differentiate first.

$$\left(\frac{\sigma_K}{K}\right)^2 = \left(\frac{\sigma_m}{m}\right)^2 + \left(2\frac{\sigma_v}{v}\right)^2.$$

With relative uncertainties 3% in mass and 2% in speed,

$$\sigma_K/K = \sqrt{0.03^2 + (2 \times 0.02)^2} = 0.05.$$

A worst-case bound would use a sum, not this quadrature. Correlations require covariance terms.



2 Practice

2.1 Choose one answer. A binomial detector has N=100 and p=0.20. Its mean and standard deviation are: [1]

- A 20 and 20
- B 20 and 4
- C 20 and 16
- D 5 and 2
- E Exactly 20 with no deviation

2.2 Choose one answer. With negligible mass uncertainty and speed uncertainty 3%, small kinetic-energy fractional uncertainty is: [1]

- A 1.5%
- B 3%
- C 6%
- D 9%
- E 12%

2.3 A Poisson count has mean 25. Its standard deviation is _____. Show your relation and units. [2]

2.4 Decide whether this claim is true. Justify it: A dimensionally consistent formula necessarily has the correct numerical coefficient and physical assumptions. [2]

3 Exam-style questions

3.1 A sample interval contains 900 counts; an independent equal-duration background interval contains 100. Estimate net counts and their standard uncertainty

. If both intervals are doubled at stable rates, what happens to relative uncertainty?[4]