

20.1 Speed distributions and statistical ensembles

Name: _____ Class: _____ Date: _____

Total: 10 marks**KEY WORDS** probability density • partition function

Objective

Normalise continuous speed densities and interpret their units; Distinguish vector means, speed means and most probable speeds; Use Boltzmann weights with degeneracy and a partition function . Original public practice; not an ETS paper.

1 Key knowledge and worked examples

A speed probability density f has $\int f(v)dv=1$ and inverse-speed units. A count density P has area N and $f=P/N$. A trapezoid rising over v_0 , flat over $2v_0$ and falling over v_0 has area $3av_0$ for height a . Normalise by area, not height.

Without bulk flow an isotropic gas has zero mean velocity but positive mean speed. Maxwell speed density is proportional to $v^2 \exp[-mv^2/(2kBT)]$. Mode, mean and rms speeds are $\sqrt{2kBT/m}$, $\sqrt{8kBT/(\pi m)}$ and $\sqrt{3kBT/m}$. The v^2 factor makes speed density zero at $v=0$.

For a continuous distribution, an exact single speed has probability zero; intervals have nonzero probabilities. This does not exclude arbitrarily slow particles or a detector's finite-width zero bin. Characteristic speeds scale as $\sqrt{T/m}$, not T .

Canonical state weights are $\exp(-E/(kBT))$; divide by their sum Z . Level probability includes degeneracy g : $g \exp(-E/(kBT))/Z$. Ground degeneracy 1 and excited degeneracy 2 at gap $kBT \ln 2$ give equal level weights. Classical Maxwell-Boltzmann differs from quantum Bose/Fermi occupations.

■ Worked method

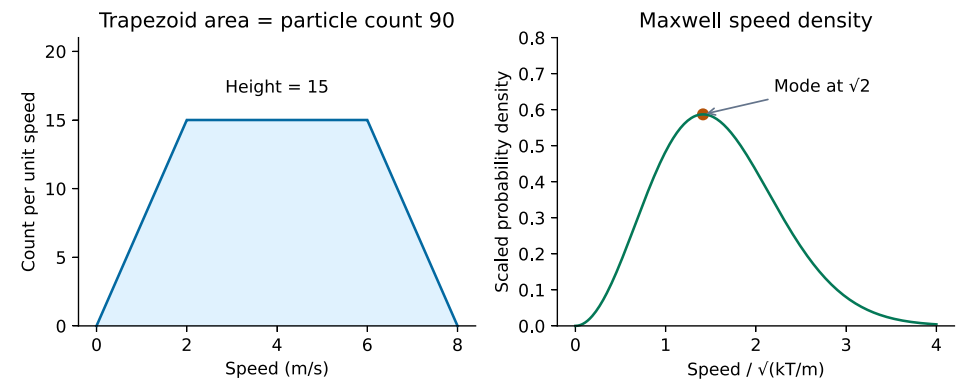
A canonical ensemble has ground degeneracy 1 and excited degeneracy g at gap Δ .

$$Z = 1 + ge^{-\Delta/(k_B T)}, \quad P_e = \frac{ge^{-\Delta/(k_B T)}}{Z}.$$

With $g = 4$ and $\Delta = k_B T \ln 2$,

$$P_e = \frac{4e^{-\ln 2}}{1 + 4e^{-\ln 2}} = \frac{2}{3}.$$

This is probability for the entire excited level. Each of its four states has probability $1/6$.



2 Practice

2.1 Choose one answer. For an isotropic equilibrium ideal gas without bulk flow, which is correct? [1]

- **A** Mean velocity and mean speed are both zero
- **B** Mean velocity is zero but mean speed is positive
- **C** The most probable speed is zero
- **D** All particles have the rms speed
- **E** Speed density has no units

2.2 Choose one answer. Ground degeneracy 1, excited degeneracy 3, gap $kBT \ln 3$. The excited-level probability is: [1]

- **A** $1/4$
- **B** $1/3$
- **C** $1/2$
- **D** $3/4$
- **E** 1

2.3 A count-density trapezoid has area $3av_0$, $N=60$ and $v_0=4$ m/s. Its height a is _____. Show your relation and units. [2]

2.4 Decide whether this claim is true. Justify it: A continuous speed distribution assigns nonzero probability to exactly one specified speed. [2]

3 Exam-style questions

3.1 Two ideal gases at the same temperature have particle masses m and $4m$. Compare their most probable, mean and rms speeds. Compare their mean translational kinetic energies and explain why zero mean velocity does not mean zero energy. [4]